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Basic solution of a Mode-I limited-permeable crack in functionally graded piezoelectric/piezomagnetic materials

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Abstract

In this paper, the basic solution of a Mode-I limited-permeable crack in functionally graded piezoelectric/piezomagnetic materials subjected to a stress loading was investigated using the generalized Almansi's theorem. In the analysis, the electric permittivity and the magnetic permeability of the air inside the crack were considered. The problem was formulated through Fourier transform into two pairs of dual integral equations about the unknown jumps of displacements across the crack surfaces. To solve the dual integral equations, the jumps of displacements across the crack surfaces were directly expanded as a series of Jacobi polynomials. The solution of the present paper shows that the singular stresses, the singular electric displacements and the singular magnetic flux at crack tips in functionally graded piezoelectric/piezomagnetic materials carry the same forms as those in homogeneous piezoelectric/piezomagnetic materials; however, the magnitudes of the magnetoelectric intensity factors depend on the gradient of functionally graded piezoelectric/piezomagnetic materials. It is also revealed that the effects of the electric and magnetic boundary conditions on the electric displacement and magnetic flux fields near crack tips can not be ignored.

Keywords: Crack; Functionally graded piezoelectric/piezomagnetic materials; Mechanics of solids

1. Introduction

From electric engineering to information technology and from ferroelectrics to piezoelectric/piezomagnetic materials, it is of great importance to study the magneto-electro-elastic interaction and fracture behaviors of magneto-electro-elastic materials (Sih and Song, 2003; Song and Sih, 2003; Wang and Mai, 2003; Gao et al., 2003a,d; Spyropoulos et al., 2003). For instances, Liu et al. (2001) studied the generalized 2D problem of an infinite magnetoelectroelastic plane with an elliptical hole using Green's

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functions; Chung and Ting (1995) obtained the two-dimensional Green functions for a magnetoelectroelastic anisotropic medium with an elliptical cavity or rigid inclusion; Pan (2002) derived the three-dimensional Green functions in anisotropic magneto-electro-elastic bimaterials; Gao et al. (2003b,c) and Wang and Mai (2004) also studied the fracture problem of piezoelectric/piezomagnetic composites by Stroh formalism; Chen et al. (2004) obtained the exact three-dimensional expressions for the full-space magneto-electro-thermo-elastic field for a penny-shaped crack subject to a uniform load on the crack surfaces with six harmonic functions; Wang and Shen (2002) obtained the general solution of three-dimensional problems in magneto-electro-elastic media through five potential functions. The development of piezoelectric/piezomagnetic composites has its roots in the early work of Van Suchtelen (1972) who proposed that the combination of piezoelectric/piezomagnetic phases may exhibit a new material property – the magnetoelectric coupling effect. Since then, only a few researchers studied magnetoelectric coupling effect in $BaTiO_3$–$CoFe_2O_4$ composites such as Wu and Huang (2000), Liu et al. (2001), Chung and Ting (1995), Pan (2002), Gao et al. (2003b,c), Wang and Mai (2004), Chen et al. (2004), Wang and Shen (2002), Harshe et al. (1993), Avellaneda and Harshe (1994), Nan (1994), Benveniste (1995), Huang and Kuo (1997), and Li (2000). Multiple-crack interaction problems in magneto-electro-elastic solids were also investigated by the 'pseudo-traction-electric displacement' method (Tian and Gabbert, 2004). Recently, Zhou and Wang (2004), Zhou et al. (2004, 2005a,b) investigated the static fracture behaviors of a single crack or two cracks in piezoelectric/piezomagnetic materials by Schmidt method (Morse and Feshbach, 1958). However, the electric permittivity and magnetic permeability of the air inside the crack in piezoelectric/piezomagnetic materials have not been considered before.

Although many experts, such as Gao et al. (1997), Hao and Shen (1994), Sosa (1992), Suo et al. (1992), Zhang and Tong (1996), Zhang et al. (1998), Zhong and Meguid (1997) and McMeeking (1989) have studied the fracture problem of piezoelectric materials, there are still arguments about the electric boundary conditions along the crack surfaces. Some authors such as Parton (1976) and Mikhailov and Parton (1990) argued that since the thickness of the crack is very small, the electric potential and the electric displacement should be continuous across the crack surfaces. This is the so-called permeable crack model. Others assumed that since air occupies the crack gap and the permittivity of the air inside the crack is far less than those of piezoelectric materials (Pak, 1990), the electric potential and the electric displacement are not continuous across the crack surface, as shown in Deeg's paper (1980). This is the so-called impermeable crack model. It was worth noting that different electric boundary conditions on the crack surfaces led to very different results (Soh et al., 2000). Strictly speaking, even if the permittivity of the air inside the crack is quite small, the flux of an electric field through the crack gap should not be zero, so it is more reasonable to assume the electric boundary condition along the crack surfaces take the following form (Hao and Shen, 1994; Hao, 2001; Dascalu and Homentcovschi, 2002):

$$D_z^+ = D_z^-, D_z^+(w^+ - w^-) = \varepsilon_0(\phi^+ - \phi^-) \tag{1}$$

in which D_z, ϕ, ε_0 and $(w^+ - w^-)$ are the electric displacement component along z-axis, the electric potential, the permittivity of the air inside the crack, and the opening displacement component of crack surfaces, respectively. This electric boundary condition was firstly proposed by Hao and Shen (1994) as the limited-permeable crack model, which will be reduced to permeable boundary conditions when $w^+ - w^- = 0$ and to impermeable ones when $\varepsilon_0 = 0$.

In this paper, the limited-permeable crack model in piezoelectric materials (Hao and Shen, 1994; Hao, 2001; Dascalu and Homentcovschi, 2002) as shown in Eq. (1) and the concept of functionally graded materials were firstly extended to deal with the electric and magnetic boundary conditions along the crack surfaces for the Mode-I fracture problem of functionally graded piezoelectric/piezomagnetic materials. The Schmidt method (Morse and Feshbach, 1958) was employed to investigate the behavior of a Mode-I crack in functionally graded piezoelectric/piezomagnetic materials subjected to a stress loading by the generalized Almansi's theorem. The Fourier transform was applied to reduce the mixed boundary-value problem to two pairs of dual integral equations about the jumps of displacements across the crack surfaces, which were solved by a direct expansion as a series of Jacobi polynomials. The solution shows that the electric and magnetic boundary conditions along the crack surfaces greatly affect the results of the electric displacement and magnetic flux fields near crack tips.

2. The Mode-I crack

It is assumed that there is a Mode-I Griffith crack of length $2l$ along x-axis in a functionally graded piezoelectric/piezomagnetic material as shown in Fig. 1.

As discussed in the literature (Hao and Shen, 1994; Hao, 2001; Dascalu and Homentcovschi, 2002), the electric permittivity and the magnetic permeability of the air inside the crack will be considered in the present study. It is assumed that a distributed normal stress loading $\sigma_{zz}(x,0) = -\tau_0(x)$. $\tau_0(x)$ is directly applied on the upper and lower crack surfaces, which is equivalent to investigating the perturbation fields for a remotely loaded cracked-body through the standard superposition technique in fracture mechanics. So the boundary conditions along the crack surfaces can be written as follows:

$$\begin{cases} \sigma_{xz}^{(1)}(x,0^+) = \sigma_{xz}^{(2)}(x,0^-) = 0, \sigma_{zz}^{(1)}(x,0^+) = \sigma_{zz}^{(2)}(x,0^-) = -\tau_0(x) \\ D_z(x,0)\left[w^{(1)}(x,0^+) - w^{(2)}(x,0^-)\right] = \varepsilon_0\left[\phi^{(1)}(x,0^+) - \phi^{(2)}(x,0^-)\right] \\ B_z(x,0)\left[w^{(1)}(x,0^+) - w^{(2)}(x,0^-)\right] = \mu_0\left[\psi^{(1)}(x,0^+) - \psi^{(2)}(x,0^-)\right] \\ D_z^{(1)}(x,0^+) = D_z^{(2)}(x,0^-), \quad B_z^{(1)}(x,0^+) = B_z^{(2)}(x,0^-) \end{cases}, \quad |x| \leqslant l \tag{2}$$

$$\begin{cases} u^{(1)}(x,0^+) = u^{(2)}(x,0^-), w^{(1)}(x,0^+) = w^{(2)}(x,0^-) \\ \sigma_{zz}^{(1)}(x,0^+) = \sigma_{zz}^{(2)}(x,0^-), \sigma_{xz}^{(1)}(x,0^+) = \sigma_{xz}^{(2)}(x,0^-) \\ \phi^{(1)}(x,0^+) = \phi^{(2)}(x,0^-) \\ \psi^{(1)}(x,0^+) = \psi^{(2)}(x,0^-) \\ D_z^{(1)}(x,0^+) = D_z^{(2)}(x,0^-) \\ B_z^{(1)}(x,0^+) = B_z^{(2)}(x,0^-) \end{cases}, \quad |x| > l \tag{3}$$

where $\sigma_{ik}^{(j)}(x,z)$, $D_k^{(j)}(x,z)$ and $B_k^{(j)}(x,z)$ ($i = x,z$, $k = x,z$, $j = 1,2$) are the plane stresses, in-plane electric displacements and in-plane magnetic fluxes; $u^{(j)}(x,z)$ and $w^{(j)}(x,z)$ are the mechanical displacement in the x- and z-directions; $\phi^{(j)}(x,z)$ and $\psi^{(j)}(x,z)$ are the electric potential and the magnetic potential, respectively. It should be noted that all quantities with superscript j ($j = 1, 2$) refers to the upper half-plane 1 and the lower half-plane 2 as shown in Fig. 1. In Eqs. (2) and (3), $D_z(x,0)$ and $B_z(x,0)$ are the electric displacement and the magnetic flux inside the crack, respectively, which depend on the external loading; ε_0 and μ_0 are the electric permittivity and the magnetic permeability of the air inside the crack, respectively. The electric and the magnetic boundary conditions expressed by Eqs. (2) and (3) will be reduced to permeable boundary condition when $w^+ - w^- = 0$ and to impermeable one when $\varepsilon_0 = 0$ and $\mu_0 = 0$.

3. Basic equations of functionally graded piezoelectric/piezomagnetic materials

For the plane problem of linear, homogeneous, transversely isotropic functionally graded piezoelectric/piezomagnetic materials with vanishing body force, the basic equations are as follows (Song and Sih, 2003; Wang and Mai, 2003; Gao et al., 2003a,d; Spyropoulos et al., 2003; Liu et al., 2001):

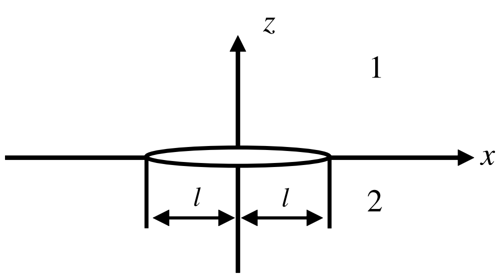

Fig. 1. The coordinate system for a crack in functionally graded piezoelectric/piezomagnetic materials.

$$\begin{cases} \frac{\partial\sigma_{xx}^{(j)}(x,z)}{\partial x}+\frac{\partial\sigma_{xz}^{(j)}(x,z)}{\partial z}=0 \\ \frac{\partial\sigma_{xz}^{(j)}(x,z)}{\partial x}+\frac{\partial\sigma_{zz}^{(j)}(x,z)}{\partial z}=0 \\ \frac{\partial D_{x}^{(j)}(x,z)}{\partial x}+\frac{\partial D_{z}^{(j)}(x,z)}{\partial z}=0 \\ \frac{\partial B_{x}^{(j)}(x,z)}{\partial x}+\frac{\partial B_{z}^{(j)}(x,z)}{\partial z}=0 \end{cases} \tag{4}$$

$$\begin{cases} \sigma_{xx}^{(j)}=c_{11}^{*}\frac{\partial u^{(j)}(x,z)}{\partial x}+c_{13}^{*}\frac{\partial w^{(j)}(x,z)}{\partial z}+e_{31}^{*}\frac{\partial\varphi^{(j)}(x,z)}{\partial z}-f_{31}^{*}\frac{\partial\psi^{(j)}(x,z)}{\partial z} \\ \sigma_{zz}^{(j)}=c_{13}^{*}\frac{\partial u^{(j)}(x,z)}{\partial x}+c_{33}^{*}\frac{\partial w^{(j)}(x,z)}{\partial z}+e_{33}^{*}\frac{\partial\varphi^{(j)}(x,z)}{\partial z}-f_{33}^{*}\frac{\partial\psi^{(j)}(x,z)}{\partial z} \\ \sigma_{xz}^{(j)}=c_{44}^{*}\left(\frac{\partial u^{(j)}(x,z)}{\partial z}+\frac{\partial w^{(j)}(x,z)}{\partial x}\right)+e_{15}^{*}\frac{\partial\varphi^{(j)}(x,z)}{\partial x}-f_{15}^{*}\frac{\partial\psi^{(j)}(x,z)}{\partial x} \\ D_{x}^{(j)}=e_{15}^{*}\left(\frac{\partial u^{(j)}(x,z)}{\partial z}+\frac{\partial w^{(j)}(x,z)}{\partial x}\right)-\varepsilon_{11}^{*}\frac{\partial\varphi^{(j)}(x,z)}{\partial x}-g_{11}^{*}\frac{\partial\psi^{(j)}(x,z)}{\partial x} \\ D_{z}^{(j)}=e_{31}^{*}\frac{\partial u^{(j)}(x,z)}{\partial x}+e_{33}^{*}\frac{\partial w^{(j)}(x,z)}{\partial z}-\varepsilon_{33}^{*}\frac{\partial\varphi^{(j)}(x,z)}{\partial z}-g_{33}^{*}\frac{\partial\psi^{(j)}(x,z)}{\partial z} \\ B_{x}^{(j)}=f_{15}^{*}\left(\frac{\partial u^{(j)}(x,z)}{\partial z}+\frac{\partial w^{(j)}(x,z)}{\partial x}\right)+g_{11}^{*}\frac{\partial\varphi^{(j)}(x,z)}{\partial x}-\mu_{11}^{*}\frac{\partial\psi^{(j)}(x,z)}{\partial x} \\ B_{z}^{(j)}=f_{31}^{*}\frac{\partial u^{(j)}(x,z)}{\partial x}+f_{33}^{*}\frac{\partial w^{(j)}(x,z)}{\partial z}+g_{33}^{*}\frac{\partial\varphi^{(j)}(x,z)}{\partial z}-\mu_{33}^{*}\frac{\partial\psi^{(j)}(x,z)}{\partial z} \end{cases} \tag{5}$$

where $c_{11}^*, c_{13}^*, c_{33}^*$ and c_{44}^* are the elastic stiffness constants; ε_{11}^* and ε_{33}^* are the dielectric constants; e_{15}^*, e_{31}^* and e_{33}^* are the piezoelectric constants; f_{15}^*, f_{31}^* and f_{33}^* are the piezomagnetic constants; g_{11}^* and g_{33}^* are the electro-magnetic constants; and μ_{11}^* and μ_{33}^* are the magnetic permeabilities.

Crack problems in functionally graded materials do not appear to be analytically tractable for arbitrary variations of material properties. Usually, one tries to generate the forms of functionally graded materials for which the problem becomes tractable. Similar to the treatment of the crack problem for isotropic functionally graded materials (Delale and Erdogan, 1988; Fildis and Yahsi, 1996; Ozturk and Erdogan, 1997; Jin and Zhong, 2002; Jin, 2003), we assume that the material properties are described by:

$$\begin{aligned} &(c_{11}^*, c_{13}^*, c_{33}^*, c_{44}^*, e_{15}^*, e_{31}^*, e_{33}^*, \varepsilon_{11}^*, \varepsilon_{33}^*, f_{15}^*, f_{31}^*, f_{33}^*, g_{11}^*, g_{33}^*) \\ &\quad = (c_{11}, c_{13}, c_{33}, c_{44}, e_{15}, e_{31}, e_{33}, \varepsilon_{11}, \varepsilon_{33}, f_{15}, f_{31}, f_{33}, g_{11}, g_{33}, \mu_{11}, \mu_{33})\mathrm{e}^{\gamma x} \end{aligned} \tag{6}$$

where γ is a constant which measures the variation rate of material properties in functionally graded materials. When $\gamma = 0$, the material properties would not change which will reduce to the homogeneous magneto-electro-elastic material case with a closed form solution reported recently (Zhou et al., 2007). The expression of Eq. (6) are purely assumed for making the problem tractable without the loss of generality.

Substituting Eqs. (5) into Eqs. (4), and using Eq. (6), the governing equations are obtained as follows:

$$\begin{aligned} &\left(c_{11}\frac{\partial^2}{\partial x^2}+c_{44}\frac{\partial^2}{\partial z^2}\right)u^{(j)}(x,z)+(c_{13}+c_{44})\frac{\partial^2}{\partial x\partial z}w^{(j)}(x,z)+(e_{31}+e_{15})\frac{\partial^2}{\partial x\partial z}\varphi^{(j)}(x,z) \\ &\qquad +(-f_{31}-f_{15})\frac{\partial^2}{\partial x\partial z}\psi^{(j)}(x,z)+\gamma\left[c_{11}\frac{\partial u^{(j)}(x,z)}{\partial x}+c_{13}\frac{\partial w^{(j)}(x,z)}{\partial z}+e_{31}\frac{\partial\varphi^{(j)}(x,z)}{\partial z}-f_{31}\frac{\partial\psi^{(j)}(x,z)}{\partial z}\right]=0 \end{aligned} \tag{7}$$

$$\begin{aligned} &(c_{13}+c_{44})\frac{\partial^2}{\partial x\partial z}u^{(j)}(x,z)+\left(c_{44}\frac{\partial^2}{\partial x^2}+c_{33}\frac{\partial^2}{\partial z^2}\right)w^{(j)}(x,z)+\left(e_{15}\frac{\partial^2}{\partial x^2}+e_{33}\frac{\partial^2}{\partial z^2}\right)\varphi^{(j)}(x,z) \\ &\qquad +\left(-f_{15}\frac{\partial^2}{\partial x^2}-f_{33}\frac{\partial^2}{\partial z^2}\right)\psi^{(j)}(x,z)+\gamma\left[c_{44}\left(\frac{\partial u^{(j)}(x,z)}{\partial z}+\frac{\partial w^{(j)}(x,z)}{\partial x}\right)\right. \\ &\qquad \left.+e_{15}\frac{\partial\varphi^{(j)}(x,z)}{\partial x}-f_{15}\frac{\partial\psi^{(j)}(x,z)}{\partial x}\right]=0 \end{aligned} \tag{8}$$

$$
\begin{aligned}
&(e_{15}+e_{31})\frac{\partial^2}{\partial x\partial z}u^{(j)}(x,z)+\left(e_{15}\frac{\partial^2}{\partial x^2}+e_{33}\frac{\partial^2}{\partial z^2}\right)w^{(j)}(x,z)+\left(-\varepsilon_{11}\frac{\partial^2}{\partial x^2}-\varepsilon_{33}\frac{\partial^2}{\partial z^2}\right)\varphi^{(j)}(x,z)\\
&\quad-\left(g_{11}\frac{\partial^2}{\partial x^2}+g_{33}\frac{\partial^2}{\partial z^2}\right)\psi^{(j)}(x,z)+\gamma\left[e_{15}\left(\frac{\partial u^{(j)}(x,z)}{\partial z}+\frac{\partial w^{(j)}(x,z)}{\partial x}\right)\right.\\
&\quad\left.-\varepsilon_{11}\frac{\partial\varphi^{(j)}(x,z)}{\partial x}-g_{11}\frac{\partial\psi^{(j)}(x,z)}{\partial x}\right]=0
\end{aligned}
\tag{9}
$$

$$
\begin{aligned}
&(f_{15}+f_{31})\frac{\partial^2}{\partial x\partial z}u^{(j)}(x,z)+\left(f_{15}\frac{\partial^2}{\partial x^2}+f_{33}\frac{\partial^2}{\partial z^2}\right)w^{(j)}(x,z)+\left(g_{11}\frac{\partial^2}{\partial x^2}+g_{33}\frac{\partial^2}{\partial z^2}\right)\varphi^{(j)}(x,z)\\
&\quad-\left(\mu_{11}\frac{\partial^2}{\partial x^2}+\mu_{33}\frac{\partial^2}{\partial z^2}\right)\psi^{(j)}(x,z)+\gamma\left[f_{15}\left(\frac{\partial u^{(j)}(x,z)}{\partial z}+\frac{\partial w^{(j)}(x,z)}{\partial x}\right)\right.\\
&\quad\left.+g_{11}\frac{\partial\varphi^{(j)}(x,z)}{\partial x}-\mu_{11}\frac{\partial\psi^{(j)}(x,z)}{\partial x}\right]=0
\end{aligned}
\tag{10}
$$

4. Solution procedures

Eqs. (7)–(10) can be solved using the method proposed by Yang (2001). First, rewrite Eqs. (7)–(10) as follows:

$$
[\mathrm{MD}]\begin{Bmatrix} u^{(j)}(x,z)\\ w^{(j)}(x,z)\\ \phi^{(j)}(x,z)\\ \psi^{(j)}(x,z)\end{Bmatrix}=0 \tag{11}
$$

where the operator matrix [MD] is

$$
[\mathrm{MD}]=\begin{bmatrix}
c_{11}\frac{\partial^2}{\partial x^2}+c_{44}\frac{\partial^2}{\partial z^2}+\gamma c_{11}\frac{\partial}{\partial x} & (c_{13}+c_{44})\frac{\partial^2}{\partial x\partial z}+\gamma c_{13}\frac{\partial}{\partial z} & (e_{31}+e_{15})\frac{\partial^2}{\partial x\partial z}+\gamma e_{31}\frac{\partial}{\partial z} & (-f_{31}-f_{15})\frac{\partial^2}{\partial x\partial z}-\gamma f_{31}\frac{\partial}{\partial z}\\
(c_{13}+c_{44})\frac{\partial^2}{\partial x\partial z}+\gamma c_{44}\frac{\partial}{\partial z} & c_{44}\frac{\partial^2}{\partial x^2}+c_{33}\frac{\partial^2}{\partial z^2}+\gamma c_{44}\frac{\partial}{\partial x} & e_{15}\frac{\partial^2}{\partial x^2}+e_{33}\frac{\partial^2}{\partial z^2}+\gamma e_{15}\frac{\partial}{\partial x} & -f_{15}\frac{\partial^2}{\partial x^2}-f_{33}\frac{\partial^2}{\partial z^2}-\gamma f_{15}\frac{\partial}{\partial x}\\
(e_{15}+e_{31})\frac{\partial^2}{\partial x\partial z}+\gamma e_{15}\frac{\partial}{\partial z} & e_{15}\frac{\partial^2}{\partial x^2}+e_{33}\frac{\partial^2}{\partial z^2}+\gamma e_{15}\frac{\partial}{\partial x} & -\varepsilon_{11}\frac{\partial^2}{\partial x^2}-\varepsilon_{33}\frac{\partial^2}{\partial z^2}-\gamma\varepsilon_{11}\frac{\partial}{\partial x} & -g_{11}\frac{\partial^2}{\partial x^2}-g_{33}\frac{\partial^2}{\partial z^2}-\gamma g_{11}\frac{\partial}{\partial x}\\
(f_{15}+f_{31})\frac{\partial^2}{\partial x\partial z}+\gamma f_{15}\frac{\partial}{\partial z} & f_{15}\frac{\partial^2}{\partial x^2}+f_{33}\frac{\partial^2}{\partial z^2}+\gamma f_{15}\frac{\partial}{\partial x} & g_{11}\frac{\partial^2}{\partial x^2}+g_{33}\frac{\partial^2}{\partial z^2}+\gamma g_{11}\frac{\partial}{\partial x} & -\mu_{11}\frac{\partial^2}{\partial x^2}-\mu_{33}\frac{\partial^2}{\partial z^2}-\gamma\mu_{11}\frac{\partial}{\partial x}
\end{bmatrix}
$$

The determinant of matrix [MD] is

$$
\begin{aligned}
\det[\mathrm{MD}]={}&d_1\frac{\partial^8}{\partial z^8}+d_2\frac{\partial^8}{\partial x^2\partial z^6}+d_3\frac{\partial^7}{\partial x\partial z^6}+d_4\frac{\partial^6}{\partial z^6}+d_5\frac{\partial^8}{\partial x^4\partial z^4}+d_6\frac{\partial^7}{\partial x^3\partial z^4}+d_7\frac{\partial^6}{\partial x^2\partial z^4}+d_8\frac{\partial^5}{\partial x\partial z^4}\\
&+d_9\frac{\partial^8}{\partial x^6\partial z^2}+d_{10}\frac{\partial^7}{\partial x^5\partial z^2}+d_{11}\frac{\partial^6}{\partial x^4\partial z^2}+d_{12}\frac{\partial^5}{\partial x^3\partial z^2}+d_{13}\frac{\partial^4}{\partial x^2\partial z^2}\\
&+d_{14}\frac{\partial^8}{\partial x^8}+d_{15}\frac{\partial^7}{\partial x^7}+d_{16}\frac{\partial^6}{\partial x^6}+d_{17}\frac{\partial^5}{\partial x^5}+d_{18}\frac{\partial^4}{\partial x^4}
\end{aligned}
$$

where d_i $(i=1,2,3,\ldots,21)$ are given in Appendix A.

Based on the cofactors Δ_{ik} of matrix [MD] (i, $k=1,2,3,4$), using the method developed in the literature (Chen et al., 2004; Yang, 2001; Ding et al., 1996), the general solution of Eq. (11) is

$$
\left[u^{(j)}(x,z),w^{(j)}(x,z),\phi^{(j)}(x,z),\psi^{(j)}(x,z)\right]^{\mathrm{T}}=(\Delta_{i1},\Delta_{i2},\Delta_{i3},\Delta_{i4})^{\mathrm{T}}\,F^{(j)}(x,z),\quad (i=1,2,3,4\text{ and }j=1,2) \tag{12}
$$

with $F^{(j)}(x,z)$ satisfying the following equation

$$\det[\mathrm{MD}]F^{(j)}(x,z)=0,\quad (j=1,2) \tag{13}$$

In the following analysis, we use only (Δ_{21}, Δ_{22}, Δ_{23}, Δ_{24}) for the present problem, which can be expressed as follows:

$$\begin{aligned}\Delta_{21}={}&\alpha_1^{(1)}\frac{\partial^3}{\partial x^2\partial z}+\alpha_2^{(1)}\frac{\partial^4}{\partial x^3\partial z}+\alpha_3^{(1)}\frac{\partial^5}{\partial x^4\partial z}+\alpha_4^{(1)}\frac{\partial^6}{\partial x^5\partial z}+\alpha_5^{(1)}\frac{\partial^4}{\partial x\partial z^3}+\alpha_6^{(1)}\frac{\partial^5}{\partial x^2\partial z^3}\\&+\alpha_7^{(1)}\frac{\partial^6}{\partial x^3\partial z^3}+\alpha_8^{(1)}\frac{\partial^5}{\partial z^5}+\alpha_9^{(1)}\frac{\partial^6}{\partial x\partial z^5}\end{aligned} \tag{14}$$

$$\begin{aligned}\Delta_{22}={}&\alpha_1^{(2)}\frac{\partial^3}{\partial x^3}+\alpha_2^{(2)}\frac{\partial^4}{\partial x^4}+\alpha_3^{(2)}\frac{\partial^5}{\partial x^5}+\alpha_4^{(2)}\frac{\partial^6}{\partial x^6}+\alpha_5^{(2)}\frac{\partial^3}{\partial x\partial z^2}+\alpha_6^{(2)}\frac{\partial^4}{\partial x^2\partial z^2}+\alpha_7^{(2)}\frac{\partial^5}{\partial x^3\partial z^2}+\alpha_8^{(2)}\frac{\partial^6}{\partial x^4\partial z^2}\\&+\alpha_9^{(2)}\frac{\partial^4}{\partial z^4}+\alpha_{10}^{(2)}\frac{\partial^5}{\partial x\partial z^4}+\alpha_{11}^{(2)}\frac{\partial^6}{\partial x^2\partial z^4}+\alpha_{12}^{(2)}\frac{\partial^6}{\partial z^6}\end{aligned} \tag{15}$$

$$\begin{aligned}\Delta_{23}={}&\alpha_1^{(3)}\frac{\partial^3}{\partial x^3}+\alpha_2^{(3)}\frac{\partial^4}{\partial x^4}+\alpha_3^{(3)}\frac{\partial^5}{\partial x^5}+\alpha_4^{(3)}\frac{\partial^6}{\partial x^6}+\alpha_5^{(3)}\frac{\partial^3}{\partial x\partial z^2}+\alpha_6^{(3)}\frac{\partial^4}{\partial x^2\partial z^2}+\alpha_7^{(3)}\frac{\partial^5}{\partial x^3\partial z^2}+\alpha_8^{(3)}\frac{\partial^6}{\partial x^4\partial z^2}\\&+\alpha_9^{(3)}\frac{\partial^4}{\partial z^4}+\alpha_{10}^{(3)}\frac{\partial^5}{\partial x\partial z^4}+\alpha_{11}^{(3)}\frac{\partial^6}{\partial x^2\partial z^4}+\alpha_{12}^{(3)}\frac{\partial^6}{\partial z^6}\end{aligned} \tag{16}$$

$$\begin{aligned}\Delta_{24}={}&\alpha_1^{(4)}\frac{\partial^3}{\partial x^3}+\alpha_2^{(4)}\frac{\partial^4}{\partial x^4}+\alpha_3^{(4)}\frac{\partial^5}{\partial x^5}+\alpha_4^{(4)}\frac{\partial^6}{\partial x^6}+\alpha_5^{(4)}\frac{\partial^3}{\partial x\partial z^2}+\alpha_6^{(4)}\frac{\partial^4}{\partial x^2\partial z^2}+\alpha_7^{(4)}\frac{\partial^5}{\partial x^3\partial z^2}+\alpha_8^{(4)}\frac{\partial^6}{\partial x^4\partial z^2}\\&+\alpha_9^{(4)}\frac{\partial^4}{\partial z^4}+\alpha_{10}^{(4)}\frac{\partial^5}{\partial x\partial z^4}+\alpha_{11}^{(4)}\frac{\partial^6}{\partial x^2\partial z^4}+\alpha_{12}^{(4)}\frac{\partial^6}{\partial z^6}\end{aligned} \tag{17}$$

where $\alpha_k^{(i)}(i=1,2,3,4;k=1,2,3,\ldots,12)$ are given in the Appendix A.

Performing Fourier transform with respect x, $F^{(j)}(x,z)$ can be expressed as follows:

$$F^{(j)}(x,z)=\frac{1}{2\pi}\int_{-\infty}^{\infty}f^{(j)}(s,z)\mathrm{e}^{\mathrm{i}sx}\,\mathrm{d}s \tag{18}$$

Substitution of Eq. (18) into Eq. (13) yields

$$a\frac{\partial^8 f^{(j)}(s,z)}{\partial z^8}-b\frac{\partial^6 f^{(j)}(s,z)}{\partial z^6}+c\frac{\partial^4 f^{(j)}(s,z)}{\partial z^4}-d\frac{\partial^2 f^{(j)}(s,z)}{\partial z^2}+ef^{(j)}(s,z)=0 \tag{19}$$

where

$$\begin{aligned}&a=d_1,\quad b=d_2s^2-\mathrm{i}d_3s-d_4,\quad c=d_5s^4-\mathrm{i}d_6s^3-d_7s^2+\mathrm{i}d_8s,\\&d=d_9s^6-\mathrm{i}d_{10}s^5-d_{11}s^4+\mathrm{i}d_{12}s^3+d_{13}s^2,\\&e=d_{14}s^8-\mathrm{i}d_{15}s^7-d_{16}s^6+\mathrm{i}d_{17}s^5+d_{18}s^4.\end{aligned}$$

Eq. (19) is a homogeneous equation, and the solution of $f^{(j)}(s,z)$ is a function of $\exp(-\lambda z)$, in which λ is the root of the following algebraic equation:

$$a\lambda^8-b\lambda^6+c\lambda^4-d\lambda^2+e=0 \tag{20}$$

which is determined by

$$\lambda_1^2 = \frac{b}{4a} - \frac{1}{2}\sqrt{R_5 + R_6} - \frac{1}{2}\sqrt{2R_5 - R_6 - \frac{\frac{b^3}{a^3} - \frac{4bc}{a^2} + \frac{8d}{a}}{4\sqrt{R_5 + R_6}}} \tag{21}$$

$$\lambda_2^2 = \frac{b}{4a} - \frac{1}{2}\sqrt{R_5 + R_6} + \frac{1}{2}\sqrt{2R_5 - R_6 - \frac{\frac{b^3}{a^3} - \frac{4bc}{a^2} + \frac{8d}{a}}{4\sqrt{R_5 + R_6}}} \tag{22}$$

$$\lambda_3^2 = \frac{b}{4a} + \frac{1}{2}\sqrt{R_5 + R_6} - \frac{1}{2}\sqrt{2R_5 - R_6 + \frac{\frac{b^3}{a^3} - \frac{4bc}{a^2} + \frac{8d}{a}}{4\sqrt{R_5 + R_6}}} \tag{23}$$

$$\lambda_4^2 = \frac{b}{4a} + \frac{1}{2}\sqrt{R_5 + R_6} + \frac{1}{2}\sqrt{2R_5 - R_6 + \frac{\frac{b^3}{a^3} - \frac{4bc}{a^2} + \frac{8d}{a}}{4\sqrt{R_5 + R_6}}} \tag{24}$$

where $R_1 = 2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace$, $R_2 = c^2 - 3bd + 12ae$, $R_3 = \sqrt{-4(R_2)^3 + (R_1)^2}$, $R_4 = \frac{(R_1+R_3)^{\frac{1}{3}}}{2^{\frac{1}{3}}}$, $R_5 = \frac{b^2}{4a^2} - \frac{2c}{3a}$, $R_6 = \frac{R_2}{3aR_4} + \frac{R_4}{3a}$.

Depending on the properties of λ^2, function $f^{(j)}(s,z)$ has five different general solutions (only for $j = 1$ and $z \geqslant 0$) (Other cases can be obtained using a similar method, but they are omitted for brevity) as follows:

(a) If $\lambda_1^2 \neq \lambda_2^2 \neq \lambda_3^2 \neq \lambda_4^2 > 0$, then

$$f^{(1)}(s,z) = A_1(s)\,\mathrm{e}^{-\lambda_1 z} + A_2(s)\,\mathrm{e}^{-\lambda_2 z} + A_3(s)\,\mathrm{e}^{-\lambda_3 z} + A_4(s)\,\mathrm{e}^{-\lambda_4 z} \tag{25}$$

(b) If $\lambda_1^2 \neq \lambda_2^2 \neq \lambda_3^2 = \lambda_4^2 > 0$, then

$$f^{(1)}(s,z) = A_1(s)\,\mathrm{e}^{-\lambda_1 z} + A_2(s)\,\mathrm{e}^{-\lambda_2 z} + A_3(s)\,\mathrm{e}^{-\lambda_3 z} + A_4(s)sz\,\mathrm{e}^{-\lambda_3 z} \tag{26}$$

(c) If $\lambda_1^2 \neq \lambda_2^2 = \lambda_3^2 = \lambda_4^2 > 0$, then

$$f^{(1)}(s,z) = A_1(s)\,\mathrm{e}^{-\lambda_1 z} + A_2(s)\,\mathrm{e}^{-\lambda_2 z} + A_3(s)sz\,\mathrm{e}^{-\lambda_2 z} + A_4(s)s^2z^2\,\mathrm{e}^{-\lambda_2 z} \tag{27}$$

(d) If $\lambda_1^2 = \lambda_2^2 = \lambda_3^2 = \lambda_4^2 > 0$, then

$$f^{(1)}(s,z) = A_1(s)\,\mathrm{e}^{-\lambda_2 z} + A_2(s)z\,\mathrm{e}^{-\lambda_2 z} + A_3(s)z^2\,\mathrm{e}^{-\lambda_2 z} + A_4(s)z^3\,\mathrm{e}^{-\lambda_2 z} \tag{28}$$

(e) If $\lambda_1^2 > 0$, $\lambda_2^2 > 0$, $\lambda_1^2 \neq \lambda_2^2$ and $\lambda_3^2, \lambda_4^2 < 0$ or λ_3^2 and λ_4^2 being a pair of conjugate complex roots, and therefore λ_3 and λ_4 are a pair of conjugate complexes $-\delta \pm \mathrm{i}\omega$, the solution of function $f^{(1)}(s,z)$ is

$$f^{(1)}(s,z) = A_1(s)\,\mathrm{e}^{-\lambda_1 z} + A_2(s)\,\mathrm{e}^{-\lambda_2 z} + A_3(s)\,\mathrm{e}^{-\delta z}\cos(\omega z) + A_4(s)\,\mathrm{e}^{-\delta z}\sin(\omega z) \tag{29}$$

where δ and $\omega > 0$ and $A_i(s)$ ($i = 1, 2, 3, 4$) is a function of s to be determined by the boundary conditions.

Substituting the solution of auxiliary function $f^{(j)}(s,z)$ into Eqs. (5), (12), (18) and Eqs. (25)–(29), the mechanical and electric displacement, stresses, electric potential fields, magnetic flux and magnetic potential fields are calculated using Mathematica. For the case of $\lambda_1^2 \neq \lambda_2^2 \neq \lambda_3^2 \neq \lambda_4^2 > 0$, the displacements, stresses, electric displacements, electric potentials, magnetic fluxes and magnetic potentials are given as follows (Other cases can be obtained using a similar method, but they are omitted in the present paper for brevity.):

$$\begin{cases} u^{(1)}(x,z) = \frac{1}{2\pi}\sum_{i=1}^{4}\int_{-\infty}^{\infty}\beta_i^{(1)}(s)A_i(s)\,\mathrm{e}^{-\lambda_i^z}\,\mathrm{e}^{\mathrm{i}sx}\,\mathrm{d}s \\ w^{(1)}(x,z) = \frac{1}{2\pi}\sum_{i=1}^{4}\int_{-\infty}^{\infty}\beta_i^{(2)}(s)A_i(s)\,\mathrm{e}^{-\lambda_i^z}\,\mathrm{e}^{\mathrm{i}sx}\,\mathrm{d}s \\ \phi^{(1)}(x,z) = \frac{1}{2\pi}\sum_{i=1}^{4}\int_{-\infty}^{\infty}\beta_i^{(3)}(s)A_i(s)\,\mathrm{e}^{-\lambda_i^z}\,\mathrm{e}^{\mathrm{i}sx}\,\mathrm{d}s \\ \psi^{(1)}(x,z) = \frac{1}{2\pi}\sum_{i=1}^{4}\int_{-\infty}^{\infty}\beta_i^{(4)}(s)A_i(s)\,\mathrm{e}^{-\lambda_i^z}\,\mathrm{e}^{\mathrm{i}sx}\,\mathrm{d}s \end{cases}, \quad z \geqslant 0 \tag{30}$$

$$\begin{cases} u^{(2)}(x,z) = -\frac{1}{2\pi}\sum_{i=1}^{4}\int_{-\infty}^{\infty}\beta_i^{(1)}(s)B_i(s)\,\mathrm{e}^{\lambda_i^z}\,\mathrm{e}^{\mathrm{i}sx}\,\mathrm{d}s \\ w^{(2)}(x,z) = \frac{1}{2\pi}\sum_{i=1}^{4}\int_{-\infty}^{\infty}\beta_i^{(2)}(s)B_i(s)\,\mathrm{e}^{\lambda_i^z}\,\mathrm{e}^{\mathrm{i}sx}\,\mathrm{d}s \\ \phi^{(2)}(x,z) = \frac{1}{2\pi}\sum_{i=1}^{4}\int_{-\infty}^{\infty}\beta_i^{(3)}(s)B_i(s)\,\mathrm{e}^{\lambda_i^z}\,\mathrm{e}^{\mathrm{i}sx}\,\mathrm{d}s \\ \psi^{(2)}(x,z) = \frac{1}{2\pi}\sum_{i=1}^{4}\int_{-\infty}^{\infty}\beta_i^{(4)}(s)B_i(s)\,\mathrm{e}^{\lambda_i^z}\,\mathrm{e}^{\mathrm{i}sx}\,\mathrm{d}s \end{cases}, \quad z \leqslant 0 \tag{31}$$

where

$$\begin{aligned} \beta_i^{(1)}(s) &= \alpha_1^{(1)}s^2\lambda_i + \alpha_2^{(1)}\mathrm{i}s^3\lambda_i - \alpha_3^{(1)}s^4\lambda_i - \alpha_4^{(1)}\mathrm{i}s^5\lambda_i - \alpha_5^{(1)}\mathrm{i}s\lambda_i^3 + \alpha_6^{(1)}s^2\lambda_i^3 + \alpha_7^{(1)}\mathrm{i}s^3\lambda_i^3 - \alpha_8^{(1)}\lambda_i^5 - \alpha_9^{(1)}\mathrm{i}s\lambda_i^5 \\ \beta_i^{(2)}(s) &= -\alpha_1^{(2)}\mathrm{i}s^3 + \alpha_2^{(2)}s^4 + \alpha_3^{(2)}\mathrm{i}s^5 - \alpha_4^{(2)}s^6 + \alpha_5^{(2)}\mathrm{i}s\lambda_i^2 - \alpha_6^{(2)}s^2\lambda_i^2 - \alpha_7^{(2)}\mathrm{i}s^3\lambda_i^2 + \alpha_8^{(2)}s^4\lambda_i^2 \\ &\quad + \alpha_9^{(2)}\lambda_i^4 + \alpha_{10}^{(2)}\mathrm{i}s\lambda_i^4 - \alpha_{11}^{(2)}s^2\lambda_i^4 + \alpha_{12}^{(2)}\lambda_i^6 \\ \beta_i^{(3)}(s) &= -\alpha_1^{(3)}\mathrm{i}s^3 + \alpha_2^{(3)}s^4 + \alpha_3^{(3)}\mathrm{i}s^5 - \alpha_4^{(3)}s^6 + \alpha_5^{(3)}\mathrm{i}s\lambda_i^2 - \alpha_6^{(3)}s^2\lambda_i^2 - \alpha_7^{(3)}\mathrm{i}s^3\lambda_i^2 + \alpha_8^{(3)}s^4\lambda_i^2 \\ &\quad + \alpha_9^{(3)}\lambda_i^4 + \alpha_{10}^{(3)}\mathrm{i}s\lambda_i^4 - \alpha_{11}^{(3)}s^2\lambda_i^4 + \alpha_{12}^{(3)}\lambda_i^6 \\ \beta_i^{(4)}(s) &= -\alpha_1^{(4)}\mathrm{i}s^3 + \alpha_2^{(4)}s^4 + \alpha_3^{(4)}\mathrm{i}s^5 - \alpha_4^{(4)}s^6 + \alpha_5^{(4)}\mathrm{i}s\lambda_i^2 - \alpha_6^{(4)}s^2\lambda_i^2 - \alpha_7^{(4)}\mathrm{i}s^3\lambda_i^2 + \alpha_8^{(4)}s^4\lambda_i^2 \\ &\quad + \alpha_9^{(4)}\lambda_i^4 + \alpha_{10}^{(4)}\mathrm{i}s\lambda_i^4 - \alpha_{11}^{(4)}s^2\lambda_i^4 + \alpha_{12}^{(4)}\lambda_i^6 \end{aligned}$$

$$\begin{cases} \sigma_{zz}^{(1)}(x,z) = \frac{\mathrm{e}^{\gamma x}}{2\pi}\sum_{i=1}^{4}\int_{-\infty}^{\infty}\chi_i^{(1)}(s)A_i(s)s\,\mathrm{e}^{-\lambda_i z}\,\mathrm{e}^{\mathrm{i}sx}\,\mathrm{d}s \\ \sigma_{xz}^{(1)}(x,z) = \frac{\mathrm{e}^{\gamma x}}{2\pi}\sum_{i=1}^{4}\int_{-\infty}^{\infty}\chi_i^{(2)}(s)A_i(s)s\,\mathrm{e}^{-\lambda_i z}\,\mathrm{e}^{\mathrm{i}sx}\,\mathrm{d}s \\ D_z^{(1)}(x,z) = \frac{\mathrm{e}^{\gamma x}}{2\pi}\sum_{i=1}^{4}\int_{-\infty}^{\infty}\chi_i^{(3)}(s)A_i(s)s\,\mathrm{e}^{-\lambda_i z}\,\mathrm{e}^{\mathrm{i}sx}\,\mathrm{d}s \\ B_z^{(1)}(x,z) = \frac{\mathrm{e}^{\gamma x}}{2\pi}\sum_{i=1}^{4}\int_{-\infty}^{\infty}\chi_i^{(4)}(s)A_i(s)s\,\mathrm{e}^{-\lambda_i z}\,\mathrm{e}^{\mathrm{i}sx}\,\mathrm{d}s \\ D_x^{(1)}(x,z) = \frac{\mathrm{e}^{\gamma x}}{2\pi}\sum_{i=1}^{4}\int_{-\infty}^{\infty}\chi_i^{(5)}(s)A_i(s)s\,\mathrm{e}^{-\lambda_i z}\,\mathrm{e}^{\mathrm{i}sx}\,\mathrm{d}s \\ B_x^{(1)}(x,z) = \frac{\mathrm{e}^{\gamma x}}{2\pi}\sum_{i=1}^{4}\int_{-\infty}^{\infty}\chi_i^{(6)}(s)A_i(s)s\,\mathrm{e}^{-\lambda_i sz}\,\mathrm{e}^{\mathrm{i}sx}\,\mathrm{d}s \end{cases}, z \geqslant 0 \tag{32}$$

$$\begin{cases} \sigma_{zz}^{(2)}(x,z) = -\frac{\mathrm{e}^{\gamma x}}{2\pi}\sum_{i=1}^{4}\int_{-\infty}^{\infty}\chi_i^{(1)}(s)B_i(s)s\,\mathrm{e}^{\lambda_i z}\,\mathrm{e}^{\mathrm{i}sx}\,\mathrm{d}s \\ \sigma_{xz}^{(2)}(x,z) = \frac{\mathrm{e}^{\gamma x}}{2\pi}\sum_{i=1}^{4}\int_{-\infty}^{\infty}\chi_i^{(2)}(s)B_i(s)s\,\mathrm{e}^{\lambda_i z}\,\mathrm{e}^{\mathrm{i}sx}\,\mathrm{d}s \\ D_z^{(2)}(x,z) = -\frac{\mathrm{e}^{\gamma x}}{2\pi}\sum_{i=1}^{4}\int_{-\infty}^{\infty}\chi_i^{(3)}(s)B_i(s)s\,\mathrm{e}^{\lambda_i z}\,\mathrm{e}^{\mathrm{i}sx}\,\mathrm{d}s \\ B_z^{(2)}(x,z) = -\frac{\mathrm{e}^{\gamma x}}{2\pi}\sum_{i=1}^{4}\int_{-\infty}^{\infty}\chi_i^{(4)}(s)B_i(s)s\,\mathrm{e}^{\lambda_i z}\,\mathrm{e}^{\mathrm{i}sx}\,\mathrm{d}s \end{cases}, z \leqslant 0 \tag{33}$$

where

$$\chi_i^{(1)}(s) = c_{13}\mathrm{i}s\beta_i^{(1)}(s) - c_{33}\lambda_i\beta_i^{(2)}(s) - e_{33}\lambda_i\beta_i^{(3)}(s) + f_{33}\lambda_i\beta_i^{(4)}(s),$$
$$\chi_i^{(2)}(s) = -c_{44}\lambda_i\beta_i^{(1)}(s) + c_{44}\mathrm{i}s\beta_i^{(2)}(s) + e_{15}\mathrm{i}s\beta_i^{(3)}(s) - f_{15}\mathrm{i}s\beta_i^{(4)}(s),$$
$$\chi_i^{(3)}(s) = e_{31}\mathrm{i}s\beta_i^{(1)}(s) - e_{33}\lambda_i\beta_i^{(2)}(s) + \varepsilon_{33}\lambda_i\beta_i^{(3)}(s) + g_{33}\lambda_i\beta_i^{(4)}(s),$$
$$\chi_i^{(4)}(s) = f_{31}\mathrm{i}s\beta_i^{(1)}(s) - f_{33}\lambda_i\beta_i^{(2)}(s) - g_{33}\lambda_i\beta_i^{(3)}(s) + \mu_{33}\lambda_i\beta_i^{(4)}(s),$$
$$\chi_i^{(5)}(s) = -e_{15}\lambda_i\beta_i^{(1)}(s) + e_{15}\mathrm{i}s\beta_i^{(2)}(s) - \varepsilon_{11}\mathrm{i}s\beta_i^{(3)}(s) - g_{11}\mathrm{i}s\beta_i^{(4)}(s),$$
$$\chi_i^{(6)}(s) = -f_{15}\lambda_i\beta_i^{(1)}(s) + f_{15}\mathrm{i}s\beta_i^{(2)}(s) + g_{11}^{\mathrm{i}}s\beta_i^{(3)}(s) - \mu_{11}\mathrm{i}s\beta_i^{(4)}(s).$$

Introduce the jumps of displacements, electric potential and magnetic potential across the crack surfaces as follows:

$$\begin{cases} f_1(x) = u^{(1)}(x,0) - u^{(2)}(x,0) \\ f_2(x) = w^{(1)}(x,0) - w^{(2)}(x,0) \\ f_3(x) = \phi^{(1)}(x,0) - \phi^{(2)}(x,0) \\ f_4(x) = \psi^{(1)}(x,0) - \psi^{(2)}(x,0) \end{cases} \tag{34}$$

we can prove that $f_1(x)$ is an odd function, $f_2(x)$, $f_3(x)$ and $f_4(x)$ are three even functions.

By substituting Eqs. (30) and (31) into Eq. (34), and the resultant equations into Eqs. (32) and (33), through Fourier transform with respect to x and using boundary conditions (2) and (3), the following equations can be obtained

$$[X_1]\begin{bmatrix} A_1(s) \\ A_2(s) \\ A_3(s) \\ A_4(s) \end{bmatrix} + [X_1]\begin{bmatrix} B_1(s) \\ B_2(s) \\ B_3(s) \\ B_4(s) \end{bmatrix} = \begin{bmatrix} \bar{f}_1(s) \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{35}$$

$$[X_2]\begin{bmatrix} A_1(s) \\ A_2(s) \\ A_3(s) \\ A_4(s) \end{bmatrix} - [X_2]\begin{bmatrix} B_1(s) \\ B_2(s) \\ B_3(s) \\ B_4(s) \end{bmatrix} = \begin{bmatrix} \bar{f}_2(s) \\ \bar{f}_3(s) \\ \bar{f}_4(s) \\ 0 \end{bmatrix} = \begin{bmatrix} \bar{f}_2(s) \\ \frac{D_0\bar{f}_2(s)}{\varepsilon_0} \\ \frac{B_0\bar{f}_2(s)}{\mu_0} \\ 0 \end{bmatrix} \tag{36}$$

where matrices $[X_i]$ $(i = 1, 2)$ are given in Appendix A.

To reveal the effects of the electric permittivity and the magnetic permeability of the air inside the crack on the intensity factors, we assumed that $D_0 = D_z(x,0)$ and $B_0 = B_z(x,0)$ ($|x| \leqslant l$) inside the crack were used as two different constants to make the problem tractable as discussed in the literature (Hao and Shen, 1994; Hao, 2001; Dascalu and Homentcovschi, 2002). Here an over bar indicates Fourier transform.

By solving the eight equations of Eq. (35) and Eq. (36) with eight unknown functions and substituting the solutions into Eq. (32) and applying the boundary conditions (2) and (3), it can be obtained (The solving processes can be shown in Appendix A.):

$$\begin{cases} \sigma_{zz}^{(1)}(x,0) = \frac{e^{\gamma x}}{2\pi}\int_{-\infty}^{\infty}\beta_1(s)\bar{f}_2(s)\,e^{\mathrm{i}sx}\,ds = -\tau_0(x) \\ \sigma_{xz}^{(1)}(x,0) = \frac{e^{\gamma x}}{2\pi}\int_{-\infty}^{\infty}\beta_2(s)\bar{f}_1(s)\,e^{\mathrm{i}sx}\,ds = 0 \end{cases}, \quad -l \leqslant x \leqslant l \tag{37}$$

$$\int_{-\infty}^{\infty}\bar{f}_1(s)\,e^{\mathrm{i}sx}\,ds = 0, \int_{-\infty}^{\infty}\bar{f}_2(s)\,e^{\mathrm{i}sx}\,ds = 0, \quad |x| > l \tag{38}$$

where $\beta_j(s)(j = 1,2)$ are known functions which are dependent on material properties as shown in Appendix A. Moreover, $\lim_{s\to+\infty}\beta_1(s)/s = \beta_1^{(0)} = -\lim_{s\to-\infty}\beta_1(s)/s$, $\lim_{s\to+\infty}\beta_2(s)/s = \beta_2^{(0)} = -\lim_{s\to-\infty}\beta_2(s)/s$ and $\beta_j^{(0)}(j = 1,2)$ are known constants which depend on material properties; D_0/ε_0 and B_0/μ_0 were given in Zhou et al. (2007) which are independent of the functionally graded parameter γ. Here, we just give these constants $\left(\beta_j^{(0)}(j = 1,2)\right)$ for $\lambda_1^2 \neq \lambda_2^2 \neq \lambda_3^2 \neq \lambda_4^2 > 0$. Other cases can be obtained using a similar method, but they are omitted for brevity. The above two pairs of dual integral equations (37) and (38) must be solved to determine the unknown functions $\bar{f}_1(s)$ and $\bar{f}_2(s)$.

5. Solution of the dual integral equations

The Schmidt method (Morse and Feshbach, 1958) is used to solve the dual integral equations (37) and (38). The jumps of displacement across the crack surfaces are directly expanded as the following series:

$$f_1(x) = \begin{cases} \sum_{n=0}^{\infty} a_n P_n^{\left(\frac{1}{2},\frac{1}{2}\right)}\left(\frac{x}{l}\right)\left(1-\frac{x^2}{l^2}\right)^{\frac{1}{2}}, & -l \leqslant x \leqslant l \\ 0, & |x| > l \end{cases} \tag{39}$$

$$f_2(x) = \begin{cases} \sum_{n=0}^{\infty} b_n P_n^{\left(\frac{1}{2},\frac{1}{2}\right)}\left(\frac{x}{l}\right)\left(1-\frac{x^2}{l^2}\right)^{\frac{1}{2}}, & -l \leqslant x \leqslant l \\ 0, & |x| > l \end{cases} \tag{40}$$

where a_n and b_n are unknown coefficients, $P_n^{(1/2,1/2)}(x)$ is a Jacobi polynomial (Gradshteyn and Ryzhik, 1980). It is worth noting that Jacobi polynomials $P_n^{(1/2,1/2)}(x)$ is equivalent to the Chebyshev polynomials $U_n(x)$, i.e., $U_n(x) = \frac{(n+1)!\sqrt{\pi}}{2\Gamma\left(n+\frac{3}{2}\right)} p_n^{\left(\frac{1}{2},\frac{1}{2}\right)}(x)$ (Abramowitz and Stegun, 1968). The Fourier transforms of Eqs. (39) and (40) are as follows(Erdelyi, 1954):

$$\bar{f}_1(s) = \sum_{n=0}^{\infty} a_n G_n \frac{1}{s} J_{n+1}(sl), G_n = 2\sqrt{\pi}(-1)^n i^n \frac{\Gamma\left(n+1+\frac{1}{2}\right)}{n!} \tag{41}$$

$$\bar{f}_2(s) = \sum_{n=0}^{\infty} b_n G_n \frac{1}{s} J_{n+1}(sl), \tag{42}$$

where $\Gamma(x)$ and $J_n(x)$ are the Gamma and Bessel functions of order n, respectively.

Substituting Eqs. (41) and (42) into Eqs. (37) and (38), it can be shown that Eq. (38) is automatically satisfied. After integration with respect to x in $[-l, x]$, Eq. (37) is reduced to the following forms:

$$\frac{1}{2\pi}\sum_{n=0}^{\infty}\int_{-\infty}^{\infty}\frac{1}{s(\mathrm{i}s+\gamma)}\beta_1(s) b_n G_n J_{n+1}(sl)\left[\mathrm{e}^{\mathrm{i}sx+\gamma x}-\mathrm{e}^{-\mathrm{i}sl-\gamma l}\right]\mathrm{d}s = -\int_{-l}^{x}\tau_0(t)\,\mathrm{d}t, -l \leqslant x \leqslant l \tag{43}$$

$$\sum_{n=0}^{\infty}\int_{-\infty}^{\infty}\frac{1}{s(\mathrm{i}s+\gamma)}\beta_2(s) a_n G_n J_{n+1}(sl)\left[\mathrm{e}^{\mathrm{i}sx+\gamma x}-\mathrm{e}^{-\mathrm{i}sl-\gamma l}\right]\mathrm{d}s = 0, -l \leqslant x \leqslant l \tag{44}$$

The semi-infinite integral in Eqs. (43) and (44) can be modified as follows:

$$\begin{aligned}
\int_{-\infty}^{\infty}\frac{\beta_j(s)}{s(\mathrm{i}s+\gamma)}J_{n+1}(sl)\left[\mathrm{e}^{\mathrm{i}sx+\gamma x}-\mathrm{e}^{-\mathrm{i}sl-\gamma l}\right]\mathrm{d}s &= \begin{cases} \frac{-2i\beta_j^{(0)}}{n+1}\left\{\mathrm{e}^{\gamma x}\cos\left[(n+1)\sin^{-1}\left(\frac{x}{l}\right)\right]-(-1)^{\frac{n+1}{2}}\mathrm{e}^{-\gamma l}\right\}, & n=1,3,5,7,\ldots \\ \frac{2\beta_j^{(0)}}{n+1}\left\{\mathrm{e}^{\gamma x}\sin\left[(n+1)\sin^{-1}\left(\frac{x}{l}\right)\right]+(-1)^{\frac{n}{2}}\mathrm{e}^{-\gamma l}\right\}, & n=0,2,4,6,\ldots \end{cases} \\
&+\int_{0}^{\infty}\frac{1}{s}\left[\frac{\beta_j(s)}{(\mathrm{i}s+\gamma)}+i\beta_j^{(0)}\right]J_{n+1}(sl)\left[\mathrm{e}^{\mathrm{i}sx+\gamma x}-\mathrm{e}^{-\mathrm{i}sl-\gamma l}\right]\mathrm{d}s \\
&+\int_{-\infty}^{0}\frac{1}{s}\left[\frac{\beta_j(s)}{(\mathrm{i}s+\gamma)}-i\beta_j^{(0)}\right]J_{n+1}(sl)\left[\mathrm{e}^{\mathrm{i}sx+\gamma x}-\mathrm{e}^{-\mathrm{i}sl-\gamma l}\right]\mathrm{d}s, \quad (j=1,2)
\end{aligned} \tag{45}$$

It can be derived that $a_n = 0 (n = 0,1,2,3,\ldots.)$, i.e. $f_1(x) = 0$.

The integrands of the semi-infinite integrals in the right hand side of Eq. (45) tend rapidly to zero for $s \to \infty$. So the semi-infinite integrals in the right hand side of Eq. (45) can be evaluated numerically. Eq. (43) can now be solved for coefficients b_n using the Schmidt method (Morse and Feshbach, 1958; Itou, 1978). It can be seen in references (Morse and Feshbach, 1958; Itou, 1978). Here, it was omitted for brevity.

6. Intensity factors

Once we determine coefficients a_n and b_n, we can obtain the stress, electric displacement and magnetic flux fields. However, from the viewpoint of fracture mechanics, it is important to determine stresses $\sigma_{zz}^{(1)}$, $\sigma_{xz}^{(1)}$, electric displacements $D_x^{(1)}$, $D_z^{(1)}$ and magnetic fluxes $B_x^{(1)}$, $B_z^{(1)}$ in the vicinity of crack tips, respectively. In the present study, $\sigma_{zz}^{(1)}$, $\sigma_{xz}^{(1)}$, $D_x^{(1)}$, $D_z^{(1)}$, $B_x^{(1)}$ and $B_z^{(1)}$ along the crack line can be expressed, respectively, as follows:

$$\begin{cases} \sigma_{zz}^{(1)}(x,0) = \frac{e^{\gamma x}}{2\pi} \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} \frac{1}{s}\beta_1(s) b_n G_n J_{n+1}(sl)\, e^{isx}\, ds \\ \sigma_{xz}^{(1)}(x,0) = \frac{e^{\gamma x}}{2\pi} \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} \frac{1}{s}\beta_2(s) a_n G_n J_{n+1}(sl)\, e^{isx}\, ds = 0 \\ D_z^{(1)}(x,0) = \frac{e^{\gamma x}}{2\pi} \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} \frac{1}{s}\beta_3(s) b_n G_n J_{n+1}(sl)\, e^{isx}\, ds \\ D_x^{(1)}(x,0) = \frac{e^{\gamma x}}{2\pi} \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} \frac{1}{s}\beta_4(s) b_n G_n J_{n+1}(sl)\, e^{isx}\, ds \\ B_z^{(1)}(x,0) = \frac{e^{\gamma x}}{2\pi} \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} \frac{1}{s}\beta_5(s) b_n G_n J_{n+1}(sl)\, e^{isx}\, ds \\ B_x^{(1)}(x,0) = \frac{e^{\gamma x}}{2\pi} \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} \frac{1}{s}\beta_6(s) b_n G_n J_{n+1}(sl)\, e^{isx}\, ds \end{cases} \tag{46}$$

where $\beta_j(s)(j = 3,4,5,6)$ are known functions which are dependent on material properties as shown in Appendix A. Moreover, $\lim_{s\to+\infty}\beta_j(s)/s = \beta_j^{(0)} = -\lim_{s\to-\infty}\beta_j(s)/s, (j = 3,4,5,6)$. $\beta_j^{(0)}(j = 3,4,5,6)$ are known constants which depend on material properties, D_0/ε_0 and B_0/μ_0 as shown in the literature (Zhou et al., 2007). They do not depend on the functionally graded parameter γ.

The singular parts of the stress, electric displacements and magnetic flux fields near the right tip of the crack in Eq. (46) can be expressed for $x > l$, respectively, as follows:

$$\begin{cases} \sigma_{zz1}^{(1)} = \frac{\beta_1^{(0)} e^{\gamma x}}{2\pi} \sum_{n=0}^{\infty} b_n G_n \left[\int_0^{\infty} J_{n+1}(sl)\, e^{isx}\, ds - \int_{-\infty}^{0} J_{n+1}(sl)\, e^{isx}\, ds\right] = \frac{\beta_1^{(0)} x}{2\pi} \sum_{n=0}^{\infty} b_n G_n Q_n(x) \\ D_{z1}^{(1)} = \frac{\beta_3^{(0)} e^{\gamma x}}{2\pi} \sum_{n=0}^{\infty} b_n G_n \left[\int_0^{\infty} J_{n+1}(sl)\, e^{isx}\, ds - \int_{-\infty}^{0} J_{n+1}(sl)\, e^{isx}\, ds\right] = \frac{\beta_3^{(0)} e^{\gamma x}}{2\pi} \sum_{n=0}^{\infty} b_n G_n Q_n(x) \\ D_{x1}^{(1)} = \frac{\beta_4^{(0)} e^{\gamma x}}{2\pi} \sum_{n=0}^{\infty} b_n G_n \left[\int_0^{\infty} J_{n+1}(sl)\, e^{isx}\, ds - \int_{-\infty}^{0} J_{n+1}(sl)\, e^{isx}\, ds\right] = \frac{\beta_4^{(0)} e^{\gamma x}}{2\pi} \sum_{n=0}^{\infty} b_n G_n Q_n(x) \\ B_{z1}^{(1)} = \frac{\beta_5^{(0)} e^{\gamma x}}{2\pi} \sum_{n=0}^{\infty} b_n G_n \left[\int_0^{\infty} J_{n+1}(sl)\, e^{isx}\, ds - \int_{-\infty}^{0} J_{n+1}(sl)\, e^{isx}\, ds\right] = \frac{\beta_5^{(0)} e^{\gamma x}}{2\pi} \sum_{n=0}^{\infty} b_n G_n Q_n(x) \\ B_{x1}^{(1)} = \frac{\beta_6^{(0)} e^{\gamma x}}{2\pi} \sum_{n=0}^{\infty} b_n G_n \left[\int_0^{\infty} J_{n+1}(sl)\, e^{isx}\, ds - \int_{-\infty}^{0} J_{n+1}(sl)\, e^{isx}\, ds\right] = \frac{\beta_6^{(0)} e^{\gamma x}}{2\pi} \sum_{n=0}^{\infty} b_n G_n Q_n(x) \end{cases} \tag{47}$$

where

$$Q_n(x) = \begin{cases} \frac{-2(-1)^{\frac{n}{2}} l^{n+1}}{\sqrt{x^2-l^2}\left[x+\sqrt{x^2-l^2}\right]^{n+1}}, & n = 0,2,4,6,\ldots \\ \frac{2i(-1)^{\frac{n+1}{2}} l^{n+1}}{\sqrt{x^2-l^2}\left[x+\sqrt{x^2-l^2}\right]^{n+1}}, & n = 1,3,5,7,\ldots \end{cases}$$

The singular parts of the stress, electric displacements and magnetic flux fields near the right tip of the crack in Eq. (46) can be expressed for $x < -l$, respectively, as follows:

$$\begin{cases} \sigma_{zz1}^{(1)} = \frac{\beta_1^{(0)}\,\mathrm{e}^{\gamma x}}{2\pi}\sum_{n=0}^{\infty} b_n G_n\left[\int_0^{\infty} J_{n+1}(sl)\,\mathrm{e}^{\mathrm{i}sx}\,\mathrm{d}s - \int_{-\infty}^{0} J_{n+1}(sl)\,\mathrm{e}^{\mathrm{i}sx}\,\mathrm{d}s\right] = \frac{\beta_1^{(0)}\,\mathrm{e}^{\gamma x}}{2\pi}\sum_{n=0}^{\infty} b_n G_n Q_n^*(x) \\ D_{z1}^{(1)} = \frac{\beta_3^{(0)}\,\mathrm{e}^{\gamma x}}{2\pi}\sum_{n=0}^{\infty} b_n G_n\left[\int_0^{\infty} J_{n+1}(sl)\,\mathrm{e}^{\mathrm{i}sx}\,\mathrm{d}s - \int_{-\infty}^{0} J_{n+1}(sl)\,\mathrm{e}^{\mathrm{i}sx}\,\mathrm{d}s\right] = \frac{\beta_3^{(0)}\,\mathrm{e}^{\gamma x}}{2\pi}\sum_{n=0}^{\infty} b_n G_n Q_n^*(x) \\ D_{x1}^{(1)} = \frac{\beta_4^{(0)}\,\mathrm{e}^{\gamma x}}{2\pi}\sum_{n=0}^{\infty} b_n G_n\left[\int_0^{\infty} J_{n+1}(sl)\,\mathrm{e}^{\mathrm{i}sx}\,\mathrm{d}s - \int_{-\infty}^{0} J_{n+1}(sl)\,\mathrm{e}^{\mathrm{i}sx}\,\mathrm{d}s\right] = \frac{\beta_4^{(0)}e\mathrm{e}^{\gamma x}}{2\pi}\sum_{n=0}^{\infty} b_n G_n Q_n^*(x) \\ B_{z1}^{(1)} = \frac{\beta_5^{(0)}\mathrm{e}^{\gamma x}}{2\pi}\sum_{n=0}^{\infty} b_n G_n\left[\int_0^{\infty} J_{n+1}(sl)\,\mathrm{e}^{\mathrm{i}sx}\,\mathrm{d}s - \int_{-\infty}^{0} J_{n+1}(sl)\,\mathrm{e}^{\mathrm{i}sx}\,\mathrm{d}s\right] = \frac{\beta_5^{(0)}\,\mathrm{e}^{\gamma x}}{2\pi}\sum_{n=0}^{\infty} b_n G_n Q_n^*(x) \\ B_{x1}^{(1)} = \frac{\beta_6^{(0)}\mathrm{e}^{\gamma x}}{2\pi}\sum_{n=0}^{\infty} b_n G_n\left[\int_0^{\infty} J_{n+1}(sl)\,\mathrm{e}^{\mathrm{i}sx}\,\mathrm{d}s - \int_{-\infty}^{0} J_{n+1}(sl)\,\mathrm{e}^{\mathrm{i}sx}\,\mathrm{d}s\right] = \frac{\beta_6^{(0)}\,\mathrm{e}^{\gamma x}}{2\pi}\sum_{n=0}^{\infty} b_n G_n Q_n^*(x) \end{cases} \tag{48}$$

where

$$Q_n^*(x) = \begin{cases} \frac{-2(-1)^{\frac{n}{2}} l^{n+1}}{\sqrt{x^2-l^2}\left[|x|+\sqrt{x^2-l^2}\right]^{n+1}}, & n = 0, 2, 4, 6, \ldots \\ \frac{-2i(-1)^{\frac{n+1}{2}} l^{n+1}}{\sqrt{x^2-l^2}\left[|x|+\sqrt{x^2-l^2}\right]^{n+1}}, & n = 1, 3, 5, 7, \ldots \end{cases}$$

From Eq. (47), the stress intensity factors, $K_{\mathrm{I}R}$, $K_{\mathrm{II}R}$, electric displacement intensity factors $K_{\mathrm{I}R}^{D}, K_{\mathrm{II}R}^{D}$ and magnetic flux intensity factors $K^{B}{}_{\mathrm{I}R}, K_{\mathrm{II}R}^{B}$ at the right tip of the crack can be expressed, respectively, as follows:

$$\begin{cases} K_{\mathrm{I}R} = \lim_{x\to l^+}\sqrt{2(x-l)}\cdot\sigma_{zz}^{(1)}(x,0) = -\frac{2\beta_1^{(0)}\,\mathrm{e}^{\gamma l}}{\sqrt{\pi l}}\sum_{n=0}^{\infty} b_n \frac{\Gamma\left(n+1+\frac{1}{2}\right)}{n!} \\ K_{\mathrm{II}R} = \lim_{x\to l^+}\sqrt{2(x-l)}\cdot\sigma_{xz}^{(1)}(x,0) = 0 \\ K_{\mathrm{I}R}^{D} = \lim_{x\to l^+}\sqrt{2(x-l)}\cdot D_z^{(1)}(x,0) = \frac{\beta_3^{(0)}}{\beta_1^{(0)}}K_{\mathrm{I}R} \\ K_{\mathrm{II}R}^{D} = \lim_{x\to l^+}\sqrt{2(x-l)}\cdot D_x^{(1)}(x,0) = \frac{\beta_4^{(0)}}{\beta_1^{(0)}}K_{\mathrm{I}R} \\ K_{\mathrm{I}R}^{B} = \lim_{x\to l^+}\mathrm{e}^{\gamma - l)}\cdot B_z^{(1)}(x,0) = \frac{\beta_5^{(0)}}{\beta_1^{(0)}}K_{\mathrm{I}R} \\ K_{\mathrm{II}R}^{B} = \lim_{x\to l^+}\sqrt{2(x-l)}\cdot B_x^{(1)}(x,0) = \frac{\beta_6^{(0)}}{\beta_1^{(0)}}K_{\mathrm{I}R} \end{cases} \tag{49}$$

From (48), the stress intensity factors $K_{\mathrm{I}L}$, $K_{\mathrm{II}L}$, electric displacement intensity factors $K_{\mathrm{I}L}^{D}, K_{\mathrm{II}L}^{D}$ and magnetic flux intensity factors $K^{B}{}_{\mathrm{I}L}, K_{\mathrm{II}L}^{B}$ at the left tip of the crack can be expressed, respectively, as follows:

$$\begin{cases} K_{\mathrm{I}L} = \lim_{x\to -l^-}\sqrt{2(x+l)}\cdot\sigma_{zz}^{(1)}(x,0) = \frac{2\beta_1^{(0)}\,\mathrm{e}^{-\gamma l}}{\sqrt{\pi l}}\sum_{n=0}^{\infty}(-1)^{n+1} b_n \frac{\Gamma\left(n+1+\frac{1}{2}\right)}{n!} \\ K_{\mathrm{II}L} = \lim_{x\to -l^-}\sqrt{2(x+l)}\cdot\sigma_{xz}^{(1)}(x,0) = 0 \\ K_{\mathrm{I}L}^{D} = \lim_{x\to -l^-}\sqrt{2(x+l)}\cdot D_z^{(1)}(x,0) = \frac{\beta_3^{(0)}}{\beta_1^{(0)}}K_{\mathrm{I}L} \\ K_{\mathrm{II}L}^{D} = \lim_{x\to -l^-}\sqrt{2(x+l)}\cdot D_x^{(1)}(x,0) = \frac{\beta_4^{(0)}}{\beta_1^{(0)}}K_{\mathrm{I}L} \\ K_{\mathrm{I}L}^{B} = \lim_{x\to -l^-}\sqrt{2(x+l)}\cdot B_z^{(1)}(x,0) = \frac{\beta_5^{(0)}}{\beta_1^{(0)}}K_{\mathrm{I}L} \\ K_{\mathrm{II}L}^{B} = \lim_{x\to -l^-}\sqrt{2(x+l)}\cdot B_x^{(1)}(x,0) = \frac{\beta_6^{(0)}}{\beta_1^{(0)}}K_{\mathrm{I}L} \end{cases} \tag{50}$$

7. Discussion and conclusions

It can be seen from previous works (Zhou and Wang, 2004; Zhou et al., 2004, 2005a,b) that the Schmidt method possesses sufficient accuracy if the first ten terms of the infinite series in Eqs. (43) and (46) are retained. Crack surface loading $-\tau_0(x)$ will simply be assumed to be a polynomial of the following form (The properties of materials are non-symmetric about z-axis for $\gamma \neq 0$, so the stress loading on the crack surfaces should not be a uniform tensile stress.):

$$-\tau_0(x) = -p_0 - p_1\left(\frac{x}{l}\right) - p_2\left(\frac{x}{l}\right)^2 - p_3\left(\frac{x}{l}\right)^3 \tag{51}$$

Since the problem is linear, the results can be obtained through superposition in any suitable manner. Here the results are obtained by taking only one of the four input parameters p_0, p_1, p_2 and p_3 as nonzero each time in the calculation. The magnetoelectroelastic material properties are listed in Table 1 (Song and Sih, 2003; Tian and Gabbert, 2004). Although the values of D_0 and B_0 depend on the external loading, we assume in the calculation that D_0 and B_0 are two constants, which allows us to focus on the effect of the electric permittivity ε_0 and the magnetic permeability μ_0 of the air inside the crack on the stress, electric displacement and magnetic flux fields near crack tips. Since ε_0 and μ_0 are two variables, D_0/ε_0 and B_0/μ_0 can be used as two variables in the calculation.

The calculated stress, electric displacement and magnetic flux intensity factors at crack tips are plotted in Figs. 2–32, respectively.

We discuss the results and draw our conclusions as follows:

(i) In the present paper, the basic solution of a Mode- Icrack in functionally graded piezoelectric/piezomagnetic materials is obtained by the generalized Almansi's theorem. This method is feasible for general cases, as discussed in Eqs. (25)–(29), and thus the obtained solution is valid in general cases. In contrast,

Table 1
Material properties of magneto-electro-elastic composite materials

c_{11} (10^{10} N/m^2)	c_{12} (10^{10} N/m^2)	c_{13} (10^{10} N/m^2)	c_{33} (10^{10} N/m^2)	c_{44} (10^{10} N/m^2)
22.6	12.5	12.4	21.6	4.4
e_{31} (C/m^2)	e_{33} (C/m^2)	e_{15} (C/m^2)	ε_{11} (10^{-10} C^2/N m^2)	ε_{33} (10^{-10} C^2/N m^2)
−2.2	9.3	5.8	56.4	63.5
f_{31} (N/Am)	f_{33} (N/Am)	f_{15} (N/Am)	μ_{11} (10^{-6} Ns^2/C^2)	μ_{33} (10^{-6} Ns^2/C^2)
290.2	350.0	275.0	297.0	83.5
g_{11} (10^{-9} Ns/VC)	g_{33} (10^{-9} Ns/VC)			
0.005	0.008			

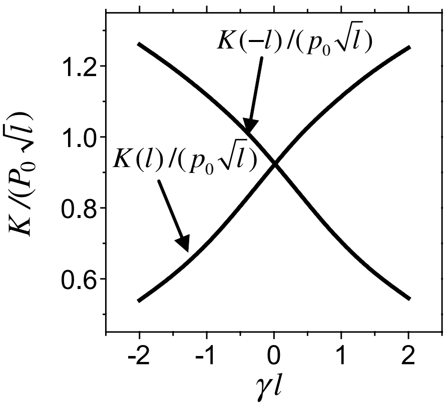

Fig. 2. Stress intensity factor versus γl for $l = 1.0$, $D_0/\varepsilon_0 = 5 \times 10^8$ and $B_0/\mu_0 = 5 \times 10^7$ ($\tau_0(x) = p_0$).

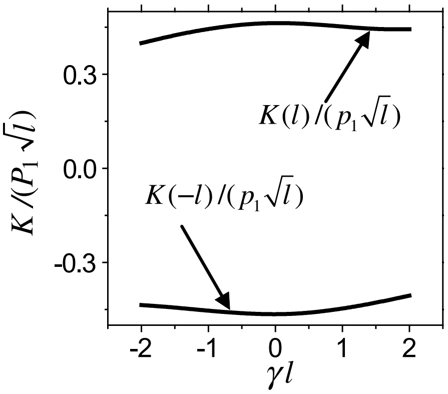

Fig. 3. Stress intensity factor versus γl for $l = 1.0$, $D_0/\varepsilon_0 = 5 \times 10^8$ and $B_0/\mu_0 = 5 \times 10^7$ ($\tau_0(x) = p_1(x/l)$).

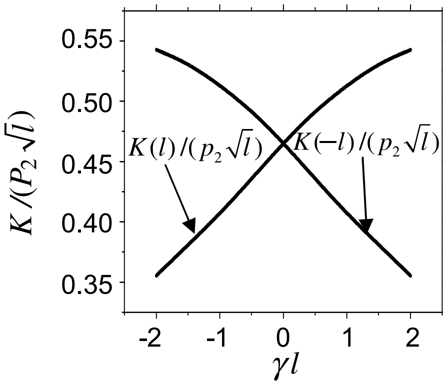

Fig. 4. Stress intensity factor versus γl for $D_0/\varepsilon_0 = 5 \times 10^8$, $l = 1.0$ and $B_0/\mu_0 = 5 \times 10^7$ ($\tau_0(x) = p_2(x/l)^2$).

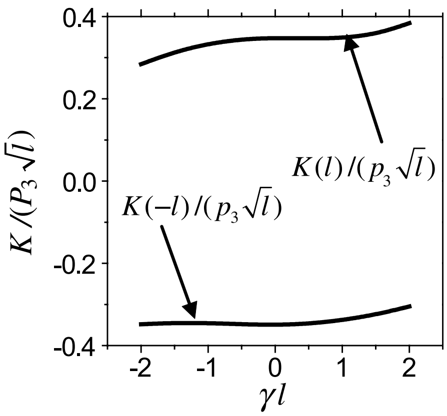

Fig. 5. Stress intensity factor versus γl for $D_0/\varepsilon_0 = 5 \times 10^8$, $l = 1.0$ and $B_0/\mu_0 = 5 \times 10^7$ ($\tau_0(x) = p_3(x/l)^3$).

the Eshelby-Stroh's method which is adopted in Gao et al. (2003b,c) is valid only for the cases of non-degenerate materials. As discussed in literature (Hao and Shen, 1994; Hao, 2001; Dascalu and Homentcovschi, 2004), the electric permittivity and the magnetic permeability of the air inside the crack are considered in the present paper to mimic the real electromagnetic boundary conditions along the crack surfaces. When $\gamma = 0$, the solution can be reverted to a closed form one for homogeneous piezoelectric/piezomagnetic materials as shown in the literature (Zhou et al., 2007).

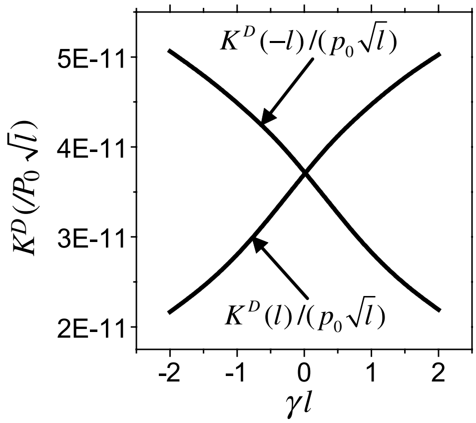

Fig. 6. Electric displacement intensity factor versus γl for $l = 1.0$, $D_0/\varepsilon_0 = 5\times10^8$ and $B_0/\mu_0 = 5\times10^7$ ($\tau_0(x) = p_0$).

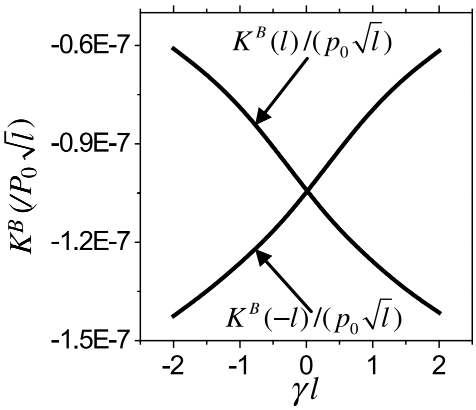

Fig. 7. Magnetic flux intensity factor versus γl for $l = 1.0$, $D_0/\varepsilon_0 = 5\times10^8$ and $B_0/\mu_0 = 5\times10^7$ ($\tau_0(x) = p_0$).

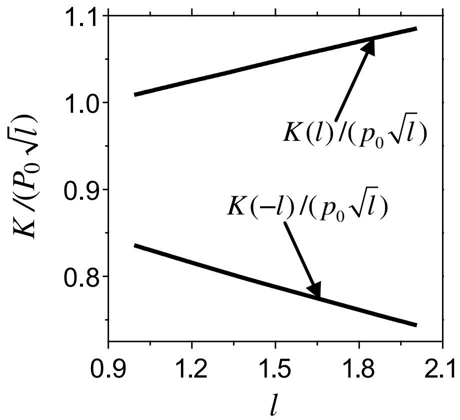

Fig. 8. Stress intensity factor versus l for $\gamma = 0.4$, $D_0/\varepsilon_0 = 5\times10^8$and $B_0/\mu_0 = 5\times10^7$ ($\tau_0(x) = p_0$).

(ii) The current paper introduces the jumps of displacements across the crack surfaces as unknown variables in constructing the dual integral equations, which is quite different from the previous work (Wu and Huang, 2000; Sih and Song, 2003; Wang and Mai, 2003; Gao et al., 2003a,d; Liu et al., 2001; Pan, 2002; Hao and Shen, 1994; Hao, 2001; Dascalu and Homentcovschi, 2002), in which the unknown vari-

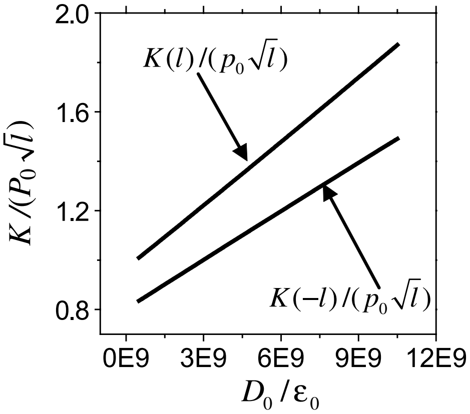

Fig. 9. Stress intensity factor versus D_0/ε_0 for $l = 1.0$, $\gamma l = 0.4$ and $B_0/\mu_0 = 5\times 10^7$ ($\tau_0(x) = p_0$).

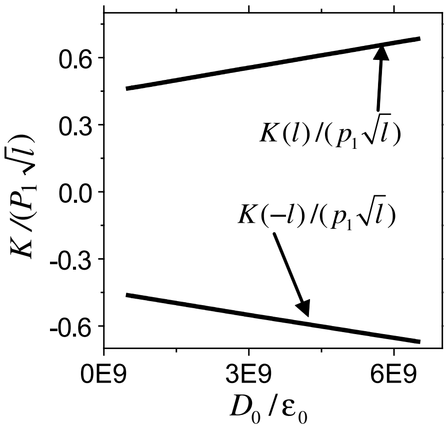

Fig. 10. Stress intensity factor versus D_0/ε_0 for $l = 1.0$, $\gamma = 0.4$ and $B_0/\mu_0 = 5\times 10^7$ ($\tau_0(x) = p_1(x/l)$).

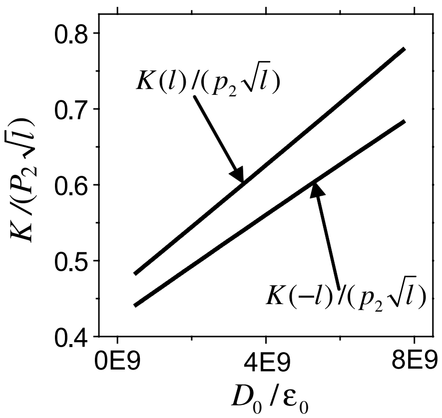

Fig. 11. Stress intensity factor versus D_0/ε_0 for $l = 1.0$, $\gamma = 0.4$ and $B_0/\mu_0 = 5\times 10^7$ ($\tau_0(x) = p_2(x/l)^2$).

ables of dual integral equations are dislocation density functions. Furthermore, to solve the dual integral equations, the jumps of displacements across the crack surfaces are directly expanded in a series of Jacobi polynomials. This is the major difference between the current work and the available work in the literature.

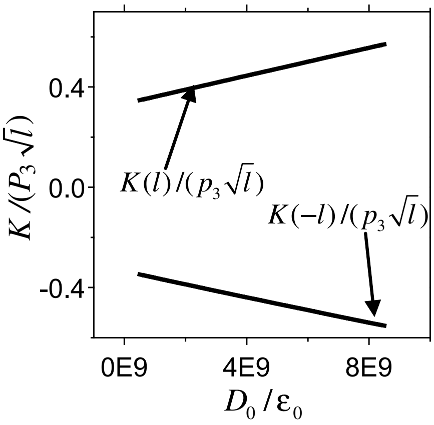

Fig. 12. Stress intensity factor versus D_0/ε_0 for $l = 1.0$, $\gamma = 0.4$ and $B_0/\mu_0 = 5 \times 10^7$ ($\tau_0\,(x) = p_3(x/l)^3$).

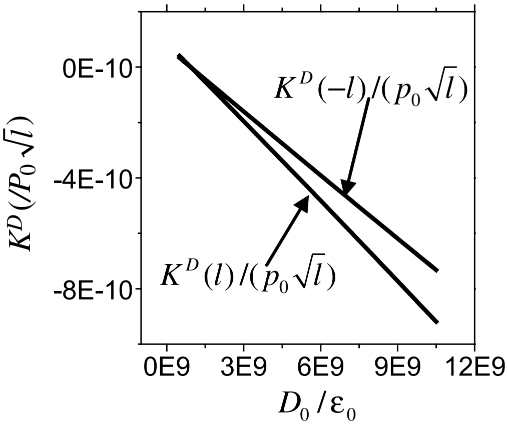

Fig. 13. Electric displacement intensity factor versus $\gamma\ l$ for $l = 1.0$, $\gamma = 0.4$ and $B_0/\mu_0 = 5 \times 10^7$ ($\tau_0(x) = p_0$).

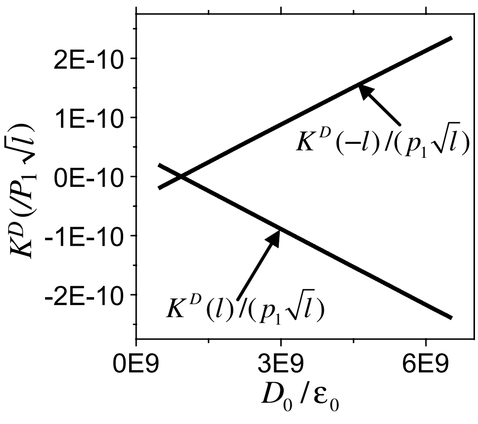

Fig. 14. Electric displacement intensity factor versus D_0/ε_0 for $l = 1.0$, $\gamma = 0.4$ and $B_0/\mu_0 = 5 \times 10^7$ ($\tau_0(x) = p_1(x/l)$).

(iii) The solution shows that the singular stress, the singular electric displacement and the singular magnetic flux in functionally graded piezoelectric/piezomagnetic materials carry the same forms as those in homogeneous piezoelectric/piezomagnetic materials, except that the magnitudes of the stress, electric displace-

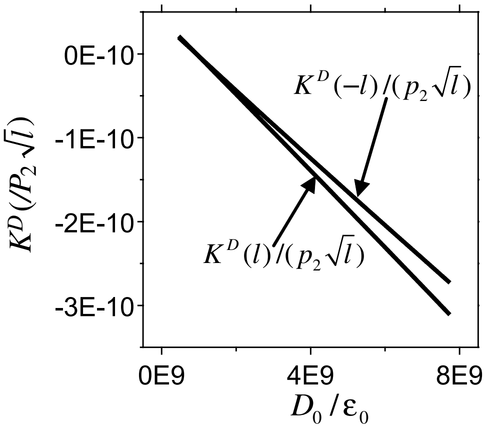

Fig. 15. Electric displacement intensity factor versus D_0/ε_0 for $l = 1.0$, $\gamma = 0.4$ and $B_0/\mu_0 = 5 \times 10^7$ ($\tau_0(x) = p_2(x/l)^2$).

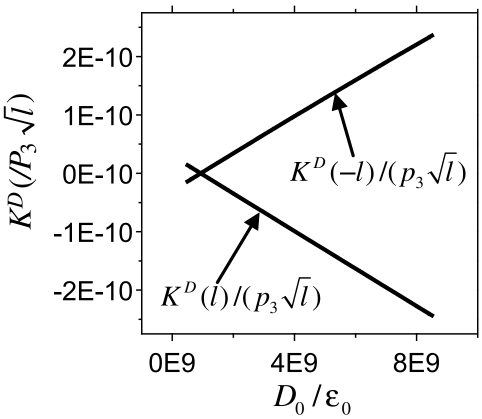

Fig. 16. Electric displacement intensity factor versus $\gamma\ l$ for $l = 1.0$, $\gamma = 0.4$ and $B_0/\mu_0 = 5 \times 10^7$ ($\tau_0(x) = p_3(x/l)^3$).

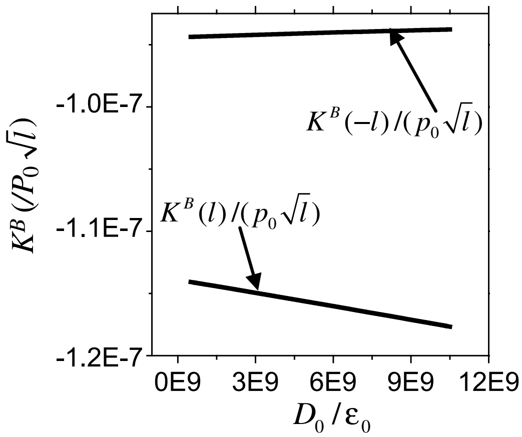

Fig. 17. Magnetic flux intensity factor versus γl for $l = 1.0$, $\gamma = 0.4$ and $B_0/\mu_0 = 5 \times 10^7$ ($\tau_0\ (x) = p_0$).

ment and magnetic flux intensity factors depend significantly upon the gradient of functionally graded piezoelectric/piezomagnetic materials as reported by Li and Weng (2002) for the fracture problem in functionally graded piezoelectric materials. The stress, electric displacement and magnetic flux intensity

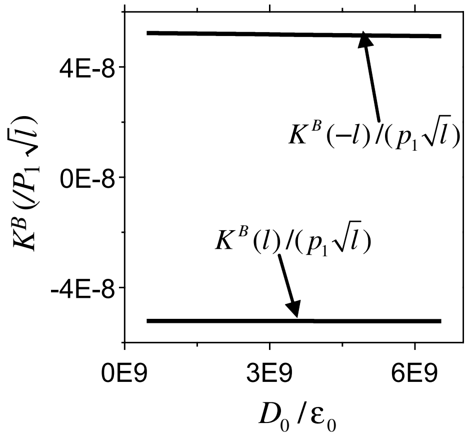

Fig. 18. Magnetic flux intensity factor versus D_0/ε_0 for $l = 1.0$, $\gamma = 0.4$and $B_0/\mu_0 = 5 \times 10^7$ ($\tau_0(x) = p_1(x/l)$).

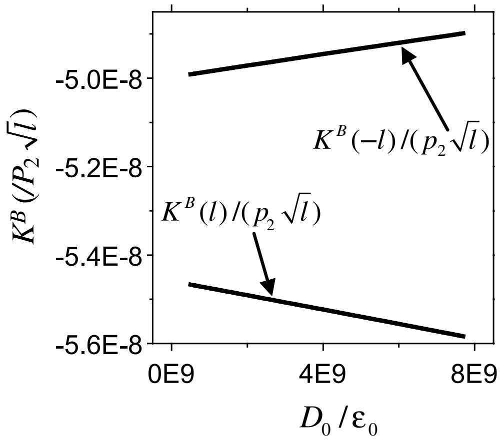

Fig. 19. Magnetic flux intensity factor versus$\gamma l$for $l = 1.0$, $\gamma = 0.4$ and $B_0/\mu_0 = 5 \times 10^7$ ($\tau_0\,(x) = p_2(x/l)^2$).

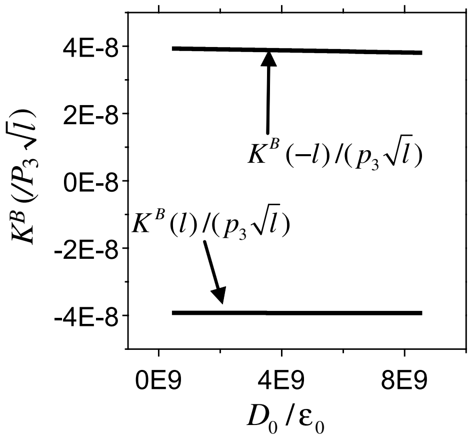

Fig. 20. Magnetic flux intensity factor versus γl for $l = 1.0$, $\gamma = 0.4$ and $B_0/\mu_0 = 5 \times 10^7$ ($\tau_0\,(x) = p_3(x/l)^3$).

factors depend on the crack length and material properties for the Mode-I crack in functionally graded piezoelectric/piezomagnetic materials as shown in Eqs. (37) and (46). The electro-magneto-elastic coupling effects on the magnetoelectroelastic intensity factors are also obtained, as shown in Eqs. (49)

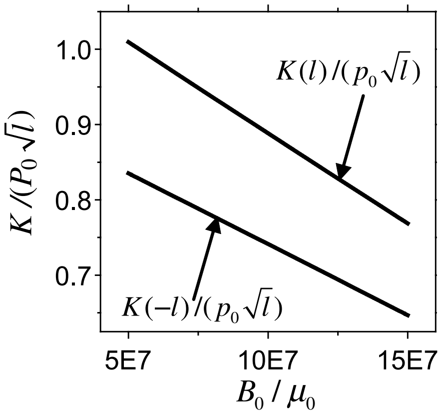

Fig. 21. Stress intensity factor versus B_0/μ_0 for $l = 1.0$, $\gamma = 0.4$ and $D_0/\varepsilon_0 = 5 \times 10^8$ ($\tau_0(x) = p_0$).

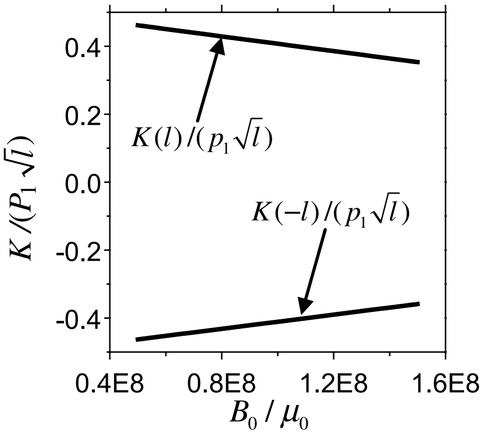

Fig. 22. Stress intensity factor versus B_0/μ_0 for $l = 1.0$, $\gamma = 0.4$ and $D_0/\varepsilon_0 = 5 \times 10^8$ ($\tau_0(x) = p_1(x/l)$).

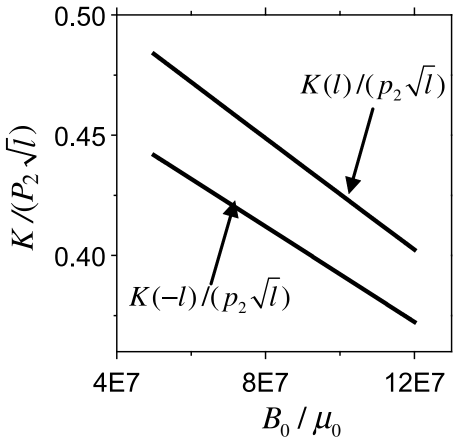

Fig. 23. Stress intensity factor versus B_0/μ_0 for $l = 1.0$, $\gamma = 0.4$ and $D_0/\varepsilon_0 = 5 \times 10^8$ ($\tau_0(x) = p_2(x/l)^2$).

and (50). The electric displacement and magnetic flux intensity factors shown in Figs. 2, 6 and 7 display the same varying tendencies as the stress intensity factors with variation in γl as shown in Eqs. (49) and (50). In addition, they are very small in magnitude as shown in Figs. 6 and 7. Therefore, some results of

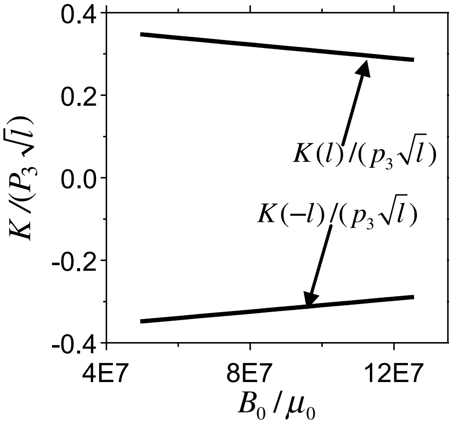

Fig. 24. Stress intensity factor versus B_0/μ_0 for $l = 1.0$, $\gamma = 0.4$ and $D_0/\varepsilon_0 = 5 \times 10^8$ ($\tau_0(x) = p_3(x/l)^3$).

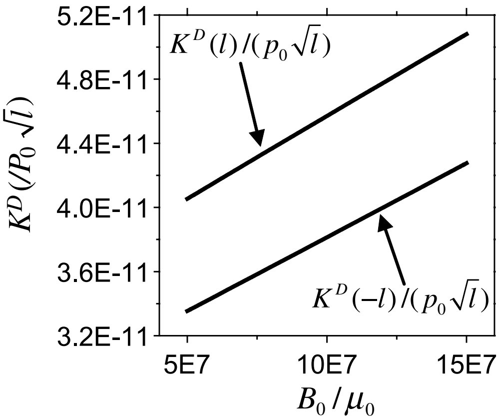

Fig. 25. Electric displacement intensity factor versus B_0/μ_0 for $l = 1.0$, $\gamma = 0.4$ and $D_0/\varepsilon_0 = 5 \times 10^8$ ($\tau_0(x) = p_0$).

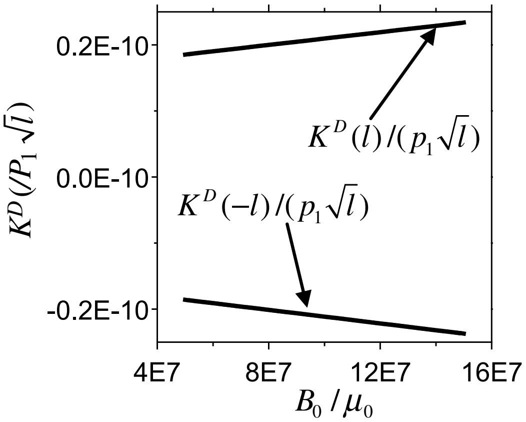

Fig. 26. Electric displacement intensity factor versus B_0/μ_0 for $l = 1.0$, $\gamma = 0.4$ and $D_0/\varepsilon_0 = 5 \times 10^8$ ($\tau_0(x) = p_1(x/l)$).

the electric displacement and magnetic flux intensity factors are omitted in the present paper for brevity. Actually, the results of electric displacement and magnetic flux intensity factors can be directly obtained from the results of stress intensity factors through Eqs. (49) and (50).

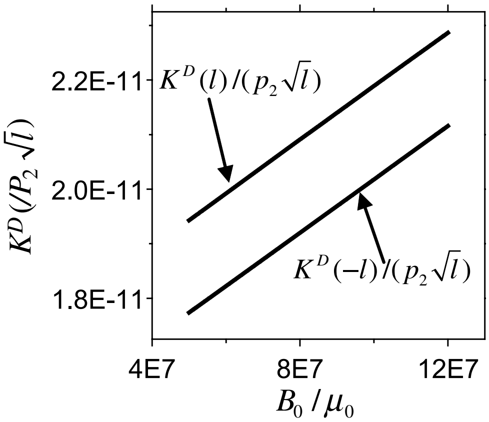

Fig. 27. Electric displacement intensity factor versus B_0/μ_0 for $l = 1.0$, $\gamma = 0.4$ and $D_0/\varepsilon_0 = 5\times10^8$ ($\tau_0(x) = p_2(x/l)^2$).

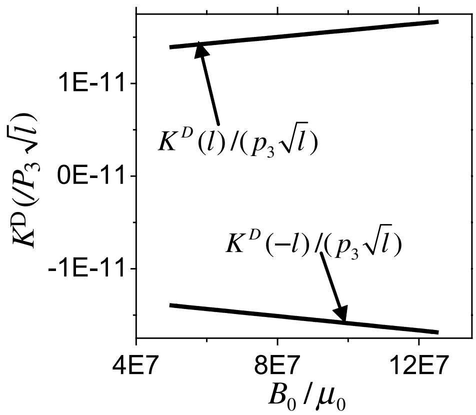

Fig. 28. Electric displacement intensity factor versus B_0/μ_0 for $l = 1.0$, $\gamma = 0.4$ and $D_0/\varepsilon_0 = 5\times10^8$ ($\tau_0(x) = p_3(x/l)^3$).

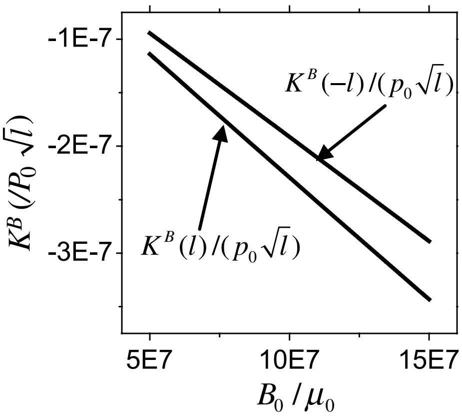

Fig. 29. Magnetic flux intensity factor versus B_0/μ_0 for $l = 1.0$, $\gamma = 0.4$ and $D_0/\varepsilon_0 = 5\times10^8$ ($\tau_0(x) = p_0$).

(iv) For symmetric loading case, the results of stress, electric displacement and magnetic flux intensity factors at crack tips are symmetric about the line, $\gamma l = 0$, as shown in Figs. 2, 4, 6 and 7. It can be also observed that $K(l)/(p_0\sqrt{l}) = K(-l)/(p_0\sqrt{l})$, $K^D(l)/(p_0\sqrt{l}) = K^B(-l)/(p_0\sqrt{l})$ and $K^B(l)/(p_0\sqrt{l}) = K^B(-l)/(p_0\sqrt{l})$ for $\gamma l = 0$ are shown in Figs. 2, 4, 6 and 7. It is the same as the closed form solution reported in Zhou et al. (2007). However, for anti-symmetric loading, the stress intensity factors are anti-symmetric about

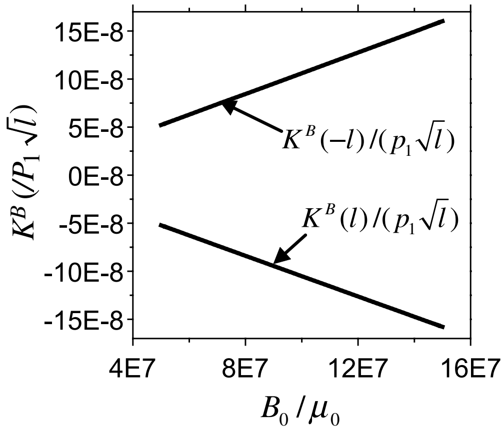

Fig. 30. Magnetic flux intensity factor versus B_0/μ_0 for $l = 1.0, \gamma = 0.4$ and $D_0/\varepsilon_0 = 5 \times 10^8$ ($\tau_0(x) = p_1(x/l)$).

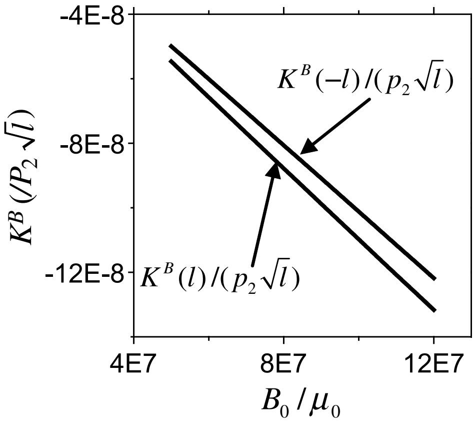

Fig. 31. Magnetic flux intensity factor versus B_0/μ_0 for $l = 1.0$, $\gamma = 0.4$ and $D_0/\varepsilon_0 = 5 \times 10^8$ ($\tau_0(x) = p_2(x/l)^2$).

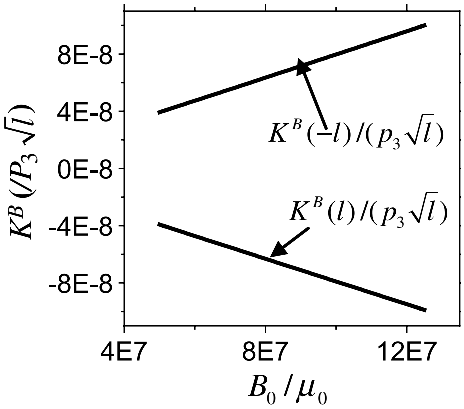

Fig. 32. Magnetic flux intensity factor versus B_0/μ_0 for $l = 1.0$, $\gamma = 0.4$ and $D_0/\varepsilon_0 = 5 \times 10^8$ ($\tau_0(x) = p_3(x/l)^3$).

the point $K = 0$ and $\gamma l = 0$ as shown in Figs. 3 and 5. So for the symmetric loading, we only discuss the properties of stress, electric displacement and magnetic flux intensity factors at the right tip of the crack here and after.

(v) For symmetric loading, the stress and electric displacement intensity factors at the right tip of the crack increase with the increase in the gradient parameter γl as shown in Figs. 2, 4 and 6. However, the stress and electric displacement intensity factors at the left tip of the crack decrease with the increase in the functionally graded parameter γl. It can be also obtained that the stress and electric displacement intensity factors at the right tip of the crack are larger than the corresponding ones at the left tip of the crack for $\gamma l > 0$; while they are less than the corresponding ones at the left tip of the crack for $\gamma l < 0$, as shown in Figs. 2, 4 and 6. This implies that the stress intensity on the stiffer side of the medium is always greater than that on the less stiff side. However, the magnitude of magnetic flux intensity factor at the right tip of the crack decreases with the increase in the gradient parameter, γl, as shown in Fig. 7. This means that, by adjusting the gradient parameter for functionally graded piezoelectric/piezomagnetic materials, the stress, electric displacement and the magnetic flux fields near crack tips can be reduced.

(vi) For anti-symmetric loading, the stress intensity factor at the right tip of the crack increases until reaching the peak value at $\gamma l = 0$, and then decreases with the increase in the gradient parameter, γl, as shown in Figs. 3 and 5. In this case, the stress intensity factor at the left tip of the crack is negative because the loading $\tau_0(x) < 0$ for $x < 0$.

(vii) As shown in Fig. 8, it can be obtained that the stress intensity factor at the right tip of the crack increases with the increase in the crack length. However, the stress intensity factor at the left tip of the crack increases with the increase in the crack length. This phenomenon may be caused by the effect of functionally graded material parameter as discussed by Shbeeb and Binienda (1999). It should be further studied in future.

(viii) For symmetric loading, the stress intensity factors at both crack tips increase with the increase in D_0/ε_0 as shown in Figs. 9 and 11. This reveals that the stress intensity factors at both crack tips increase with the decrease in the electric permittivity ε_0 of the air inside the crack. For the anti-symmetric loading, the magnitudes of stress intensity factors at crack tips increase with the increase in D_0/ε_0 as shown in Figs. 10 and 12. This reveals, however, that the magnitudes of stress intensity factors at both crack tips increase with the decrease in the electric permittivity ε_0 of the air inside the crack. Therefore, we conclude that the effects of the electric permittivity ε_0 of air inside the crack on the near tip stress fields cannot be ignored.

(ix) For symmetric or anti-symmetric loading, the magnitudes of electric displacement intensity factors at both crack tips increase linearly with the increase in D_0/ε_0 as shown in Figs. 13–16. For the anti-symmetric loading, it can be also obtained that the electric displacement intensity factors at both crack tips are symmetric about the line $K^D = 0.0$, as shown in Figs. 14 and 16.

(x) For symmetric or anti-symmetric loading as shown in Figs. 17–20, the magnitudes of magnetic flux intensity factors at the right crack tip increase linearly with the increase of D_0/ε_0. However, the magnitudes of magnetic flux intensity factors at the left crack tip decrease linearly with the increase of D_0/ε_0. It can be also obtained that the effects of the electric permittivity ε_0 of the air in the crack on electric displacement and magnetic flux fields near crack tips can not be ignored from solution of the present paper.

(xi) For symmetric or anti-symmetric loading as shown in Figs. 21–24, the magnitudes of stress intensity factors at both crack tips decrease linearly with the increase in B_0/μ_0. This reveals that the stress intensity factors at the crack tips increase with the increase in the magnetic permeability, μ_0, of the air inside the crack. We also conclude that the effects of the magnetic permeability, μ_0, of the air inside the crack on the near tip stress fields cannot be ignored.

(xii) For symmetric or anti-symmetric loading as shown in Figs. 25–28, the magnitudes of electric displacement intensity factors at both crack tips increase linearly with the increase in B_0/μ_0. This indicates that the electric displacement intensity factors at the crack tips increase with the decrease in the magnetic permeability, μ_0, of the air inside the crack.

(xiii) For symmetric or anti-symmetric loading as shown in Figs. 29–32, the magnitudes of the magnetic flux intensity factors at both crack tips increase linearly with the increase in B_0/μ_0. This indicates that the magnetic flux intensity factors at the crack tips increase with the decrease in the magnetic permeability, μ_0, of the air inside the crack. We also conclude that the effects of the magnetic permeability, μ_0, of the air inside the crack on the electric displacement and magnetic flux fields near the crack tips cannot be ignored.

Acknowledgments

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Appendix A. Coefficients

$$
\begin{aligned}
d_1 &= c_{44}(-2e_{33}f_{33}g_{33} + c_{33}g_{33}^2 + e_{33}^2\mu_{33} - f_{33}^2\varepsilon_{33} + c_{33}\mu_{33}\varepsilon_{33}),\\
d_2 &= -\{e_{31}^2f_{33}^2 + 2c_{33}e_{31}f_{15}g_{33} + 2c_{33}e_{31}f_{31}g_{33} - 2c_{13}e_{31}f_{33}g_{33} - 2c_{44}e_{31}f_{33}g_{33} - 2c_{33}c_{44}g_{11}g_{33}\\
&\quad + c_{13}^2g_{33}^2 - c_{11}c_{33}g_{33}^2 + 2c_{13}c_{44}g_{33}^2 - c_{33}e_{31}^2\mu_{33} + e_{33}^2(f_{15}^2 + 2f_{15}f_{31} + f_{31}^2 - c_{44}\mu_{11} - c_{11}\mu_{33})\\
&\quad + e_{15}^2(f_{33}^2 - c_{33}\mu_{33}) + 2e_{15}\{[c_{33}(f_{15} + f_{31}) - c_{13}f_{33}]g_{33} + e_{31}(f_{33}^2 - c_{33}\mu_{33})\}\\
&\quad - 2e_{33}\{-c_{44}f_{33}g_{11} + c_{13}f_{15}g_{33} + c_{13}f_{31}g_{33} + c_{44}f_{31}g_{33} - c_{11}f_{33}g_{33}\\
&\quad + e_{15}(f_{15}f_{33} + f_{31}f_{33} - c_{13}\mu_{33}) + e_{31}[f_{15}f_{33} + f_{31}f_{33} - \mu_{33}(c_{13} + c_{44})]\}\\
&\quad + c_{44}f_{33}^2\varepsilon_{11} - c_{33}c_{44}\mu_{33}\varepsilon_{11} + c_{33}f_{15}^2\varepsilon_{33} + 2c_{33}f_{15}f_{31}\varepsilon_{33} + c_{33}f_{31}^2\varepsilon_{33} - 2c_{13}f_{15}f_{33}\varepsilon_{33} - 2c_{13}f_{31}f_{33}\varepsilon_{33}\\
&\quad - 2c_{44}f_{31}f_{33}\varepsilon_{33} + c_{11}f_{33}^2\varepsilon_{33} - c_{33}c_{44}\mu_{11}\varepsilon_{33} + c_{13}^2\mu_{33}\varepsilon_{33} - c_{11}c_{33}\mu_{33}\varepsilon_{33} + 2c_{13}c_{44}\mu_{33}\varepsilon_{33}\},
\end{aligned}
$$

$$
\begin{aligned}
d_3 &= -\gamma\{e_{31}^2f_{33}^2 + 2c_{33}e_{31}f_{15}g_{33} + 2c_{33}e_{31}f_{31}g_{33} - 2c_{13}e_{31}f_{33}g_{33} - 2c_{44}e_{31}f_{33}g_{33} - c_{33}c_{44}g_{11}g_{33}\\
&\quad + c_{13}^2g_{33}^2 - c_{11}c_{33}g_{33}^2 + 2c_{13}c_{44}g_{33}^2 - c_{33}e_{31}^2\mu_{33} + e_{33}^2(f_{15}^2 + 2f_{15}f_{31} + f_{31}^2 - c_{44}\mu_{11} - c_{11}\mu_{33})\\
&\quad + e_{15}^2(f_{33}^2 - c_{33}\mu_{33}) + 2e_{15}g_{33}(c_{33}f_{15} + c_{33}f_{31} - c_{13}f_{33}) + 2e_{15}e_{31}(f_{33}^2 - c_{33}\mu_{33})\\
&\quad + e_{33}[c_{44}f_{33}g_{11} - 2c_{13}f_{15}g_{33} - 2c_{13}f_{31}g_{33} - 2c_{44}f_{31}g_{33} + 2c_{11}f_{33}g_{33}\\
&\quad - 2e_{15}(f_{15}f_{33} + f_{31}f_{33} - c_{13}\mu_{33}) - 2e_{31}(f_{15}f_{33} + f_{31}f_{33} - c_{13}\mu_{33} - c_{44}\mu_{33})]\\
&\quad + c_{44}f_{33}^2\varepsilon_{11} - c_{33}c_{44}\mu_{33}\varepsilon_{11} + c_{33}f_{15}^2\varepsilon_{33} + 2c_{33}f_{15}f_{31}\varepsilon_{33} + c_{33}f_{31}^2\varepsilon_{33} - 2c_{13}f_{15}f_{33}\varepsilon_{33} - 2c_{13}f_{31}f_{33}\varepsilon_{33}\\
&\quad - 2c_{44}f_{31}f_{33}\varepsilon_{33} + c_{11}f_{33}^2\varepsilon_{33} - c_{33}c_{44}\mu_{11}\varepsilon_{33} + c_{13}^2\mu_{33}\varepsilon_{33} - c_{11}c_{33}\mu_{33}\varepsilon_{33} + 2c_{13}c_{44}\mu_{33}\varepsilon_{33}\},
\end{aligned}
$$

$$
\begin{aligned}
d_4 &= \gamma^2[-e_{33}^2f_{15}f_{31} - c_{33}e_{31}f_{15}g_{33} + c_{44}e_{31}f_{33}g_{33} - c_{13}c_{44}g_{33}^2 - c_{33}f_{15}f_{31}\varepsilon_{33} + c_{13}f_{15}f_{33}\varepsilon_{33} + c_{44}f_{31}f_{33}\varepsilon_{33}\\
&\quad - c_{13}c_{44}\mu_{33}\varepsilon_{33} + e_{33}(e_{31}f_{15}f_{33} + e_{15}f_{31}f_{33} + c_{13}f_{15}g_{33} + c_{44}f_{31}g_{33} - c_{13}e_{15}\mu_{33} - c_{44}e_{31}\mu_{33})\\
&\quad + e_{15}(-e_{31}f_{33}^2 - c_{33}f_{31}g_{33} + c_{13}f_{33}g_{33} + c_{33}e_{31}\mu_{33})]
\end{aligned}
$$

$$
\begin{aligned}
d_5 &= 2c_{13}e_{33}f_{15}g_{11} + 2c_{13}e_{33}f_{31}g_{11} + 2c_{44}e_{33}f_{31}g_{11} - 2c_{11}e_{33}f_{33}g_{11} + c_{33}c_{44}g_{11}^2 - 2c_{11}e_{33}f_{15}g_{33}\\
&\quad - 2c_{13}^2g_{11}g_{33} + 2c_{11}c_{33}g_{11}g_{33} - 4c_{13}c_{44}g_{11}g_{33} + c_{11}c_{44}g_{33}^2 + c_{11}e_{33}^2\mu_{11}\\
&\quad + e_{15}^2(2f_{31}f_{33} + c_{33}\mu_{11} - 2c_{13}\mu_{33}) + e_{31}^2(-2f_{15}f_{33} + c_{33}\mu_{11} + c_{44}\mu_{33})\\
&\quad - 2e_{15}[c_{33}(f_{15} + f_{31})g_{11} - c_{13}f_{33}g_{11} - 2c_{13}f_{15}g_{33} - c_{13}f_{31}g_{33} + c_{11}f_{33}g_{33}\\
&\quad + e_{33}(f_{15}f_{31} + f_{31}^2 + c_{13}\mu_{11} - c_{11}\mu_{33})]\\
&\quad + 2e_{31}\{-c_{33}f_{15}g_{11} - c_{33}f_{31}g_{11} + c_{13}f_{33}g_{11} + c_{44}f_{33}g_{11} + c_{13}f_{15}g_{33} - c_{44}f_{31}g_{33}\\
&\quad + e_{33}[f_{15}^2 + f_{15}f_{31} - (c_{13} + c_{44})\mu_{11}] + e_{15}(-f_{15}f_{33} + f_{31}f_{33} + c_{33}\mu_{11} - c_{13}\mu_{33})\}\\
&\quad - c_{33}f_{15}^2\varepsilon_{11} - 2c_{33}f_{15}f_{31}\varepsilon_{11} - c_{33}f_{31}^2\varepsilon_{11} + 2c_{13}f_{15}f_{33}\varepsilon_{11} + 2c_{13}f_{31}f_{33}\varepsilon_{11} + 2c_{44}f_{31}f_{33}\varepsilon_{11}\\
&\quad - c_{11}f_{33}^2\varepsilon_{11} + c_{33}c_{44}\mu_{11}\varepsilon_{11} - c_{13}^2\mu_{33}\varepsilon_{11} + c_{11}c_{33}\mu_{33}\varepsilon_{11} - 2c_{13}c_{44}\mu_{33}\varepsilon_{11} + 2c_{13}f_{15}^2\varepsilon_{33}\\
&\quad + 2c_{13}f_{15}f_{31}\varepsilon_{33} - c_{44}f_{31}^2\varepsilon_{33} - 2c_{11}f_{15}f_{33}\varepsilon_{33} - c_{13}^2\mu_{11}\varepsilon_{33} + c_{11}c_{33}\mu_{11}\varepsilon_{33} - 2c_{13}c_{44}\mu_{11}\varepsilon_{33} + c_{11}c_{44}\mu_{33}\varepsilon_{33},
\end{aligned}
$$

$$
\begin{aligned}
d_6 = 2\gamma\{&2c_{13}e_{33}f_{15}g_{11} + 2c_{13}e_{33}f_{31}g_{11} + 2c_{44}e_{33}f_{31}g_{11} - 2c_{11}e_{33}f_{33}g_{11} + c_{33}c_{44}g_{11}^2 - 2c_{11}e_{33}f_{15}g_{33} \\
&- 2c_{13}^2g_{11}g_{33} + 2c_{11}c_{33}g_{11}g_{33} - 4c_{13}c_{44}g_{11}g_{33} + c_{11}c_{44}g_{33}^2 + c_{11}e_{33}^2\mu_{11} \\
&+ e_{15}^2(2f_{31}f_{33} + c_{33}\mu_{11} - 2c_{13}\mu_{33}) + e_{31}^2(-2f_{15}f_{33} + c_{33}\mu_{11} + c_{44}\mu_{33}) \\
&- 2e_{15}[c_{33}(f_{15} + f_{31})g_{11} - c_{13}f_{33}g_{11} - 2c_{13}f_{15}g_{33} - c_{13}f_{31}g_{33} + c_{11}f_{33}g_{33} \\
&+ e_{33}(f_{15}f_{31} + f_{31}^2 + c_{13}\mu_{11} - c_{11}\mu_{33})] \\
&+ 2e_{31}[-c_{33}f_{15}g_{11} - c_{33}f_{31}g_{11} + c_{13}f_{33}g_{11} + c_{44}f_{33}g_{11} + c_{13}f_{15}g_{33} - c_{44}f_{31}g_{33} \\
&+ e_{33}(f_{15}^2 + f_{15}f_{31} - c_{13}\mu_{11} - c_{44}\mu_{11}) + e_{15}(-f_{15}f_{33} + f_{31}f_{33} + c_{33}\mu_{11} - c_{13}\mu_{33})] \\
&- c_{33}f_{15}^2\varepsilon_{11} - 2c_{33}f_{15}f_{31}\varepsilon_{11} - c_{33}f_{31}^2\varepsilon_{11} + 2c_{13}f_{15}f_{33}\varepsilon_{11} + 2c_{13}f_{31}f_{33}\varepsilon_{11} + 2c_{44}f_{31}f_{33}\varepsilon_{11} - c_{11}f_{33}^2\varepsilon_{11} \\
&+ c_{33}c_{44}\mu_{11}\varepsilon_{11} - c_{13}^2\mu_{33}\varepsilon_{11} + c_{11}c_{33}\mu_{33}\varepsilon_{11} - 2c_{13}c_{44}\mu_{33}\varepsilon_{11} + 2c_{13}f_{15}^2\varepsilon_{33} + 2c_{13}f_{15}f_{31}\varepsilon_{33} - c_{44}f_{31}^2\varepsilon_{33} \\
&- 2c_{11}f_{15}f_{33}\varepsilon_{33} - c_{13}^2\mu_{11}\varepsilon_{33} + c_{11}c_{33}\mu_{11}\varepsilon_{33} - 2c_{13}c_{44}\mu_{11}\varepsilon_{33} + c_{11}c_{44}\mu_{33}\varepsilon_{33}\},
\end{aligned}
$$

$$
\begin{aligned}
d_7 = \gamma^2\{&3c_{13}e_{33}f_{15}g_{11} + 2c_{13}e_{33}f_{31}g_{11} + 3c_{44}e_{33}f_{31}g_{11} - 2c_{11}e_{33}f_{33}g_{11} + c_{33}c_{44}g_{11}^2 - 2c_{11}e_{33}f_{15}g_{33} \\
&- 2c_{13}^2g_{11}g_{33} + 2c_{11}c_{33}g_{11}g_{33} - 6c_{13}c_{44}g_{11}g_{33} + c_{11}c_{44}g_{33}^2 + c_{11}e_{33}^2\mu_{11} + e_{15}^2(3f_{31}f_{33} + c_{33}\mu_{11} - 3c_{13}\mu_{33}) \\
&+ e_{31}^2(-2f_{15}f_{33} + c_{33}\mu_{11} + c_{44}\mu_{33}) - e_{15}(2c_{33}f_{15}g_{11} + 3c_{33}f_{31}g_{11} - 3c_{13}f_{33}g_{11} - 6c_{13}f_{15}g_{33} \\
&- 2c_{13}f_{31}g_{33} + 2c_{11}f_{33}g_{33} + 3e_{33}f_{15}f_{31} + 2e_{33}f_{31}^2 + 3e_{33}c_{13}\mu_{11} - 2e_{33}c_{11}\mu_{33}) \\
&+ e_{31}[-3c_{33}f_{15}g_{11} - 2c_{33}f_{31}g_{11} + 2c_{13}f_{33}g_{11} + 3c_{44}f_{33}g_{11} + 2c_{13}f_{15}g_{33} - 2c_{44}f_{31}g_{33} \\
&+ e_{33}(3f_{15}^2 + 2f_{15}f_{31} - 2c_{13}\mu_{11} - 3c_{44}\mu_{11}) + e_{15}(-3f_{15}f_{33} + 2f_{31}f_{33} + 3c_{33}\mu_{11} - 2c_{13}\mu_{33})] \\
&- c_{33}f_{15}^2\varepsilon_{11} - 3c_{33}f_{15}f_{31}\varepsilon_{11} - c_{33}f_{31}^2\varepsilon_{11} + 3c_{13}f_{15}f_{33}\varepsilon_{11} + 2c_{13}f_{31}f_{33}\varepsilon_{11} + 3c_{44}f_{31}f_{33}\varepsilon_{11} - c_{11}f_{33}^2\varepsilon_{11} \\
&+ c_{33}c_{44}\mu_{11}\varepsilon_{11} - c_{13}^2\mu_{33}\varepsilon_{11} + c_{11}c_{33}\mu_{33}\varepsilon_{11} - 3c_{13}c_{44}\mu_{33}\varepsilon_{11} + 3c_{13}f_{15}^2\varepsilon_{33} + 2c_{13}f_{15}f_{31}\varepsilon_{33} - c_{44}f_{31}^2\varepsilon_{33} \\
&- 2c_{11}f_{15}f_{33}\varepsilon_{33} - c_{13}^2\mu_{11}\varepsilon_{33} + c_{11}c_{33}\mu_{11}\varepsilon_{33} - 3c_{13}c_{44}\mu_{11}\varepsilon_{33} + c_{11}c_{44}\mu_{33}\varepsilon_{33}\},
\end{aligned}
$$

$$
\begin{aligned}
d_8 = \gamma^3[&c_{13}e_{33}f_{15}g_{11} + c_{44}e_{33}f_{31}g_{11} - 2c_{13}c_{44}g_{11}g_{33} - e_{15}(e_{33}f_{15}f_{31} + c_{33}f_{31}g_{11} - c_{13}f_{33}g_{11} - 2c_{13}f_{15}g_{33} \\
&+ c_{13}e_{33}\mu_{11}) + e_{31}(-e_{15}f_{15}f_{33} - c_{33}f_{15}g_{11} + c_{44}f_{33}g_{11} + c_{33}e_{15}\mu_{11} + e_{33}f_{15}^2 - e_{33}c_{44}\mu_{11}) \\
&+ e_{15}^2f_{31}f_{33} - e_{15}^2c_{13}\mu_{33} - c_{33}f_{15}f_{31}\varepsilon_{11} + c_{13}f_{15}f_{33}\varepsilon_{11} + c_{44}f_{31}f_{33}\varepsilon_{11} - c_{13}c_{44}\mu_{33}\varepsilon_{11} + c_{13}f_{15}^2\varepsilon_{33} - c_{13}c_{44}\mu_{11}\varepsilon_{33}],
\end{aligned}
$$

$$
\begin{aligned}
d_9 = &-2c_{11}e_{33}f_{15}g_{11} - c_{13}^2g_{11}^2 + c_{11}c_{33}g_{11}^2 - 2c_{13}c_{44}g_{11}^2 + 2c_{11}c_{44}g_{11}g_{33} + e_{31}^2(-f_{15}^2 + c_{44}\mu_{11}) \\
&+ 2e_{31}(e_{15}f_{15}f_{31} + c_{13}f_{15}g_{11} - c_{44}f_{31}g_{11} - c_{13}e_{15}\mu_{11}) \\
&+ 2e_{15}[c_{13}(2f_{15} + f_{31})g_{11} - c_{11}(f_{33}g_{11} + f_{15}g_{33} - e_{33}\mu_{11})] \\
&- e_{15}^2(f_{31}^2 + 2c_{13}\mu_{11} - c_{11}\mu_{33}) + 2c_{13}f_{15}^2\varepsilon_{11} + 2c_{13}f_{15}f_{31}\varepsilon_{11} - c_{44}f_{31}^2\varepsilon_{11} - 2c_{11}f_{15}f_{33}\varepsilon_{11} \\
&- c_{13}^2\mu_{11}\varepsilon_{11} + c_{11}c_{33}\mu_{11}\varepsilon_{11} - 2c_{13}c_{44}\mu_{11}\varepsilon_{11} + c_{11}c_{44}\mu_{33}\varepsilon_{11} - c_{11}f_{15}^2\varepsilon_{33} + c_{11}c_{44}\mu_{11}\varepsilon_{33},
\end{aligned}
$$

$$
\begin{aligned}
d_{10} = 3\gamma\{&-2c_{11}e_{33}f_{15}g_{11} - c_{13}^2g_{11}^2 + c_{11}c_{33}g_{11}^2 - 2c_{13}c_{44}g_{11}^2 + 2c_{11}c_{44}g_{11}g_{33} + e_{31}^2(-f_{15}^2 + c_{44}\mu_{11}) \\
&+ 2e_{31}(e_{15}f_{15}f_{31} + c_{13}f_{15}g_{11} - c_{44}f_{31}g_{11} - c_{13}e_{15}\mu_{11}) \\
&+ 2e_{15}[c_{13}(2f_{15} + f_{31})g_{11} - c_{11}(f_{33}g_{11} + f_{15}g_{33} - e_{33}\mu_{11})] \\
&- e_{15}^2(f_{31}^2 + 2c_{13}\mu_{11} - c_{11}\mu_{33}) + 2c_{13}f_{15}^2\varepsilon_{11} + 2c_{13}f_{15}f_{31}\varepsilon_{11} - c_{44}f_{31}^2\varepsilon_{11} - 2c_{11}f_{15}f_{33}\varepsilon_{11} \\
&- c_{13}^2\mu_{11}\varepsilon_{11} + c_{11}c_{33}\mu_{11}\varepsilon_{11} - 2c_{13}c_{44}\mu_{11}\varepsilon_{11} + c_{11}c_{44}\mu_{33}\varepsilon_{11} - c_{11}f_{15}^2\varepsilon_{33} + c_{11}c_{44}\mu_{11}\varepsilon_{33}\},
\end{aligned}
$$

$$
\begin{aligned}
d_{11} = \gamma^2\{&-6c_{11}e_{33}f_{15}g_{11} - 3c_{13}^2g_{11}^2 + 3c_{11}c_{33}g_{11}^2 - 7c_{13}c_{44}g_{11}^2 + 6c_{11}c_{44}g_{11}g_{33} - 3e_{31}^2f_{15}^2 + 3e_{31}^2c_{44}\mu_{11} \\
&+ 6e_{31}(e_{15}f_{15}f_{31} + c_{13}f_{15}g_{11} - c_{44}f_{31}g_{11} - c_{13}e_{15}\varepsilon_{11}) + 2e_{15}[c_{13}(7f_{15} + 3f_{31})g_{11} \\
&- 3c_{11}(f_{33}g_{11} + f_{15}g_{33} - e_{33}\mu_{11})] + e_{15}^2(-3f_{31}^2 - 7c_{13}\mu_{11} + 3c_{11}\mu_{33}) \\
&+ 7c_{13}f_{15}^2\varepsilon_{11} + 6c_{13}f_{15}f_{31}\varepsilon_{11} - 3c_{44}f_{31}^2\varepsilon_{11} - 6c_{11}f_{15}f_{33}\varepsilon_{11} - 3c_{13}^2\mu_{11}\varepsilon_{11} + 3c_{11}c_{33}\mu_{11}\varepsilon_{11} - 7c_{13}c_{44}\mu_{11}\varepsilon_{11} \\
&+ 3c_{11}c_{44}\mu_{33}\varepsilon_{11} - 3c_{11}f_{15}^2\varepsilon_{33} + 3c_{11}c_{44}\mu_{11}\varepsilon_{11}\},
\end{aligned}
$$

$$
\begin{aligned}
d_{12} = {} & \gamma^3\{-2c_{11}e_{33}f_{15}g_{11} - c_{13}^2 g_{11}^2 + c_{11}c_{33}g_{11}^2 - 4c_{13}c_{44}g_{11}^2 + 2c_{11}c_{44}g_{11}g_{33} + e_{31}^2(-f_{15}^2 + c_{44}\mu_{11}) \\
& + 2e_{13}(e_{15}f_{15}f_{31} + c_{13}f_{15}g_{11} - c_{44}f_{31}g_{11} - c_{13}e_{15}\mu_{11}) + 2e_{15}[c_{13}(4f_{15} + f_{31})g_{11} - c_{11}(f_{33}g_{11} + f_{15}g_{33} - e_{33}\mu_{11})] \\
& - e_{15}^2(f_{31}^2 + 4c_{13}\mu_{11} - c_{11}\mu_{33}) + 4c_{13}f_{15}^2\varepsilon_{11} + 2c_{13}f_{15}f_{31}\varepsilon_{11} - c_{44}f_{31}^2\varepsilon_{11} - 2c_{11}f_{15}f_{33}\varepsilon_{11} - 2c_{13}^2\mu_{11}\varepsilon_{11} \\
& + c_{11}c_{33}\mu_{11}\varepsilon_{11} - 4c_{13}c_{44}\mu_{11}\varepsilon_{11} + c_{11}c_{44}\mu_{33}\varepsilon_{11} - c_{11}f_{15}^2\varepsilon_{33} + c_{11}c_{44}\mu_{11}\varepsilon_{33}\}
\end{aligned}
$$

$$
\begin{aligned}
d_{13} &= -c_{13}\gamma^4[-2e_{15}f_{15}g_{11} + e_{15}^2\mu_{11} - f_{15}^2\varepsilon_{11} + c_{44}(g_{11}^2 + \mu_{11}\varepsilon_{11})], \\
d_{14} &= -2c_{11}e_{15}f_{15}g_{11} + c_{11}c_{44}g_{11}^2 + c_{11}e_{15}^2\mu_{11} - c_{11}f_{15}^2\varepsilon_{11} + c_{11}c_{44}\mu_{11}\varepsilon_{11}, \\
d_{15} &= \gamma(-8c_{11}e_{15}f_{15}g_{11} + 4c_{11}c_{44}g_{11}^2 + 4c_{11}e_{15}^2\mu_{11} - 4c_{11}f_{15}^2\varepsilon_{11} + 4c_{11}c_{44}\mu_{11}\varepsilon_{11}), \\
d_{16} &= c_{11}\gamma^2(-12e_{15}f_{15}g_{11} + 6c_{44}g_{11}^2 + 6e_{15}^2\mu_{11} - 6f_{15}^2\varepsilon_{11} + 6c_{44}\mu_{11}\varepsilon_{11}), \\
d_{17} &= 4c_{11}\gamma^3(-2e_{15}f_{15}g_{11} + c_{44}g_{11}^2 + e_{15}^2\mu_{11} - f_{15}^2\varepsilon_{11} + c_{44}\mu_{11}\varepsilon_{11}), \\
d_{18} &= c_{11}\gamma^4(-2e_{15}f_{15}g_{11} + c_{44}g_{11}^2 + e_{15}^2\mu_{11} - f_{15}^2\varepsilon_{11} + c_{44}\mu_{11}\varepsilon_{11}), \\
\alpha_1^{(1)} &= -\gamma^3(-e_{31}f_{15}g_{11} - e_{15}f_{31}g_{11} + c_{13}g_{11}^2 + e_{15}e_{31}\mu_{11} - f_{15}f_{31}\varepsilon_{11} + c_{13}\mu_{11}\varepsilon_{11}), \\
\alpha_2^{(1)} &= -\gamma^2[-3e_{31}f_{15}g_{11} + 3c_{13}g_{11}^2 + c_{44}g_{11}^2 + e_{15}^2\mu_{11} + e_{15}(-2f_{15}g_{11} - 3f_{31}g_{11} + 3e_{31}\mu_{11}) - f_{15}^2\varepsilon_{11} \\
&\quad - 3f_{15}f_{31}\varepsilon_{11} + 3c_{13}\mu_{11}\varepsilon_{11} + c_{44}\mu_{11}\varepsilon_{11}], \\
\alpha_3^{(1)} &= -\gamma[-3e_{31}f_{15}g_{11} + 3c_{13}g_{11}^2 + 2c_{44}g_{11}^2 + 2e_{15}^2\mu_{11} + e_{15}(-4f_{15}g_{11} - 3f_{31}g_{11} + 3e_{31}\mu_{11}) - 2f_{15}^2\varepsilon_{11} \\
&\quad - f_{15}f_{31}\varepsilon_{11} + 3c_{13}\mu_{11}\varepsilon_{11} + 2c_{44}\mu_{11}\varepsilon_{11}], \\
\alpha_4^{(1)} &= -[-e_{31}f_{15}g_{11} + c_{13}g_{11}^2 + c_{44}g_{11}^2 + e_{15}^2\mu_{11} + e_{15}(-2f_{15}g_{11} - f_{31}g_{11} + e_{31}\mu_{11}) - f_{15}^2\varepsilon_{11} - f_{15}f_{31}\varepsilon_{11} \\
&\quad + c_{13}\mu_{11}\varepsilon_{11} + c_{44}\mu_{11}\varepsilon_{11}], \\
\alpha_5^{(1)} &= \gamma^2(e_{33}f_{15}g_{11} + e_{31}f_{33}g_{11} + e_{31}f_{15}g_{33} + e_{15}f_{31}g_{33} - 2c_{13}g_{11}g_{33} - e_{31}e_{33}\mu_{11} - e_{15}e_{31}\mu_{33} + f_{31}f_{33}\varepsilon_{11} \\
&\quad - c_{13}\mu_{33}\varepsilon_{11} + f_{15}f_{31}\varepsilon_{33} - c_{13}\mu_{11}\varepsilon_{33}), \\
\alpha_6^{(1)} &= -\gamma(-2e_{31}f_{33}g_{11} - 2e_{31}f_{15}g_{33} + 4c_{13}g_{11}g_{33} + 2c_{44}g_{11}g_{33} - e_{33}f_{15}g_{11} - 2e_{33}f_{31}g_{11} + e_{33}e_{15}\mu_{11} \\
&\quad + 2e_{33}e_{31}\mu_{11} + e_{15}^2\mu_{33} - e_{15}f_{33}g_{11} - 2e_{15}f_{15}g_{33} - 2e_{15}f_{31}g_{33} + 2e_{15}e_{31}\mu_{33} - f_{15}f_{33}\varepsilon_{11} - 2f_{31}f_{33}\varepsilon_{11} \\
&\quad + 2c_{13}\mu_{33}\varepsilon_{11} + c_{44}\mu_{33}\varepsilon_{11} - f_{15}^2\varepsilon_{33} - 2f_{15}f_{31}\varepsilon_{33} + 2c_{13}\mu_{11}\varepsilon_{33} + c_{44}\mu_{11}\varepsilon_{33}), \\
\alpha_7^{(1)} &= e_{31}f_{33}g_{11} + e_{31}f_{15}g_{33} - 2c_{13}g_{11}g_{33} - 2c_{44}g_{11}g_{33} + e_{33}[f_{15}g_{11} + f_{31}g_{11} - (e_{15} + e_{31})\mu_{11}] - e_{15}^2\mu_{33} \\
&\quad + e_{15}(f_{33}g_{11} + 2f_{15}g_{33} + f_{31}g_{33} - e_{31}\mu_{33}) + f_{15}f_{33}\varepsilon_{11} + f_{31}f_{33}\varepsilon_{11} - c_{13}\mu_{33}\varepsilon_{11} - c_{44}\mu_{33}\varepsilon_{11} + f_{15}^2\varepsilon_{33} \\
&\quad + f_{15}f_{31}\varepsilon_{33} - c_{13}\mu_{11}\varepsilon_{33} - c_{44}\mu_{11}\varepsilon_{33}, \\
\alpha_8^{(1)} &= -\gamma(-e_{33}f_{31}g_{33} - e_{31}f_{33}g_{33} + c_{13}g_{33}^2 + e_{31}e_{33}\mu_{33} - f_{31}f_{33}\varepsilon_{33} + c_{13}\mu_{33}\varepsilon_{33}) \\
\alpha_9^{(1)} &= -\{-e_{15}f_{33}g_{33} - e_{31}f_{33}g_{33} + c_{13}g_{33}^2 + c_{44}g_{33}^2 + e_{33}[-f_{15}g_{33} - f_{31}g_{33} + (e_{15} + e_{31})\mu_{33}] - f_{15}f_{33}\varepsilon_{33} \\
&\quad - f_{31}f_{33}\varepsilon_{33} + c_{13}\mu_{33}\varepsilon_{33} + c_{44}\mu_{33}\varepsilon_{33}\}, \\
\alpha_1^{(2)} &= c_{11}\gamma^3(g_{11}^2 + \mu_{11}\varepsilon_{11}\gamma), \\
\alpha_2^{(2)} &= 3c_{11}\gamma^2(g_{11}^2 + \mu_{11}\varepsilon_{11}), \\
\alpha_3^{(2)} &= 3c_{11}\gamma(g_{11}^2 + \mu_{11}\varepsilon_{11}\gamma), \\
\alpha_4^{(2)} &= c_{11}(g_{11}^2 + \mu_{11}\varepsilon_{11}), \\
\alpha_5^{(2)} &= \gamma^3(-e_{31}f_{15}g_{11} - e_{15}f_{31}g_{11} + e_{15}e_{31}\mu_{11} - f_{15}f_{31}\varepsilon_{11}), \\
\alpha_6^{(2)} &= \gamma^2(-2e_{15}f_{15}g_{11} - 3e_{31}f_{15}g_{11} - 3e_{15}f_{31}g_{11} - 2e_{31}f_{31}g_{11} + c_{44}g_{11}^2 + 2c_{11}g_{11}g_{33} + e_{15}^2\mu_{11} + 3e_{15}e_{31}\mu_{11} \\
&\quad + e_{31}^2\mu_{11} - f_{15}^2\varepsilon_{11} - 3f_{15}f_{31}\varepsilon_{11} - f_{31}^2\varepsilon_{11} + c_{44}\mu_{11}g_{11} + c_{11}\mu_{33}\varepsilon_{11} + c_{11}\mu_{11}\varepsilon_{33}),
\end{aligned}
$$

$$
\begin{aligned}
\alpha_7^{(2)} &= \gamma(-4e_{15}f_{15}g_{11} - 4e_{31}f_{15}g_{11} - 4e_{15}f_{31}g_{11} - 4e_{31}f_{31}g_{11} + 2c_{44}g_{11}^2 + 4c_{11}g_{11}g_{33} + 2e_{15}^2\mu_{11} \\
&\quad + 4e_{15}e_{31}\mu_{11} + 2e_{31}^2\mu_{11} - 2f_{15}^2\varepsilon_{11} - 4f_{15}f_{31}\varepsilon_{11} - 2f_{31}^2\varepsilon_{11} + 2c_{44}\mu_{11}\varepsilon_{11} + 2c_{11}\mu_{33}\varepsilon_{11} + 2c_{11}\mu_{11}\varepsilon_{33}) \\
\alpha_8^{(2)} &= -2e_{31}(f_{15} + f_{31})g_{11} + c_{44}g_{11}^2 + 2c_{11}g_{11}g_{33} + e_{15}^2\mu_{11} + e_{31}^2\mu_{11} - 2e_{15}(f_{15}g_{11} + f_{31}g_{11} - e_{31}\mu_{11}) - f_{15}^2\varepsilon_{11} \\
&\quad - 2f_{15}f_{31}\varepsilon_{11} - f_{31}^2\varepsilon_{11} + c_{44}\mu_{11}\varepsilon_{11} + c_{11}\mu_{33}\varepsilon_{11} + c_{11}\mu_{11}\varepsilon_{33}, \\
\alpha_9^{(2)} &= \gamma^2(-e_{31}f_{15}g_{33} - e_{15}f_{31}g_{33} + e_{15}e_{31}\mu_{33} - f_{15}f_{31}\varepsilon_{33}) \\
\alpha_{10}^{(2)} &= \gamma(-2e_{15}f_{15}g_{33} - 2e_{31}f_{15}g_{33} - 2e_{15}f_{31}g_{33} - 2e_{31}f_{31}g_{33} + 2c_{44}g_{11}g_{33} + c_{11}g_{33}^2 + e_{15}^2\mu_{33} + 2e_{15}e_{31}\mu_{33} \\
&\quad + e_{31}^2\mu_{33} + c_{44}\mu_{33}\varepsilon_{11} - f_{15}^2\varepsilon_{33} - 2f_{15}f_{31}\varepsilon_{33} - f_{31}^2\varepsilon_{33} + c_{44}\mu_{11}\varepsilon_{33} + c_{11}\mu_{33}\varepsilon_{33}), \\
\alpha_{11}^{(2)} &= -2e_{31}(f_{15} + f_{31})g_{33} + 2c_{44}g_{11}g_{33} + c_{11}g_{33}^2 + e_{15}^2\mu_{33} + e_{31}^2\mu_{33} - 2e_{15}(f_{15}g_{33} + f_{31}g_{33} - e_{31}\mu_{33}) \\
&\quad + c_{44}\mu_{33}\varepsilon_{11} - f_{15}^2\varepsilon_{33} - 2f_{15}f_{31}\varepsilon_{33} - f_{31}^2\varepsilon_{33} + c_{44}\mu_{11}\varepsilon_{33} + c_{11}\mu_{33}\varepsilon_{33}, \\
\alpha_{12}^{(2)} &= c_{44}(g_{33}^2 + \mu_{33}\varepsilon_{33}), \alpha_1^{(3)} = -\gamma^3c_{11}(f_{15}g_{11} - e_{15}\mu_{11}), \alpha_2^{(3)} = -3\gamma^2c_{11}(f_{15}g_{11} - e_{15}\mu_{11}), \\
\alpha_3^{(3)} &= -3\gamma c_{11}(f_{15}g_{11} - e_{15}\mu_{11}), \alpha_4^{(3)} = -c_{11}(f_{15}g_{11} - e_{15}\mu_{11}), \alpha_5^{(3)} = -\gamma^3c_{13}(-f_{15}g_{11} + e_{15}\mu_{11}), \\
\alpha_6^{(3)} &= -\gamma^2(-e_{31}f_{15}f_{31} + e_{15}f_{31}^2 - 3c_{13}f_{15}g_{11} - c_{13}f_{31}g_{11} + c_{11}f_{33}g_{11} + c_{11}f_{15}g_{33} + 3c_{13}e_{15}\mu_{11} + c_{13}e_{31}\mu_{11} \\
&\quad - c_{11}e_{33}\mu_{11} - c_{11}e_{15}\mu_{33}), \\
\alpha_7^{(3)} &= -\gamma(-e_{31}f_{15}^2 + e_{15}f_{15}f_{31} - 2e_{31}f_{15}f_{31} + 2e_{15}f_{31}^2 - 3c_{13}f_{15}g_{11} - 2c_{13}f_{31}g_{11} - c_{44}f_{31}g_{11} + 2c_{11}f_{33}g_{11} \\
&\quad + 2c_{11}f_{15}g_{33} + 3c_{13}e_{15}\mu_{11} + 2c_{13}e_{31}\mu_{11} + c_{44}e_{31}\mu_{11} - 2c_{11}e_{33}\mu_{11} - 2c_{11}e_{15}\mu_{33}), \\
\alpha_8^{(3)} &= -\{-c_{13}f_{15}g_{11} - c_{13}f_{31}g_{11} - c_{44}f_{31}g_{11} + c_{11}f_{33}g_{11} + c_{11}f_{15}g_{33} - c_{11}e_{33}\mu_{11} + e_{31}[-f_{15}^2 - f_{15}f_{31} + (c_{13} \\
&\quad + c_{44})\mu_{11}] + e_{15}(f_{15}f_{31} + f_{31}^2 + c_{13}\mu_{11} - c_{11}\mu_{33})\}, \\
\alpha_9^{(3)} &= -\gamma^2(e_{33}f_{15}f_{31} - e_{15}f_{31}f_{33} - c_{13}f_{15}g_{33} + c_{13}e_{15}\mu_{33}), \\
\alpha_{10}^{(3)} &= -\gamma(e_{33}f_{15}^2 + 2e_{33}f_{15}f_{31} + e_{33}f_{31}^2 - e_{15}f_{15}f_{33} - 2e_{15}f_{31}f_{33} - e_{31}f_{31}f_{33} + c_{44}f_{33}g_{11} - 2c_{13}f_{15}g_{33} - c_{13}f_{31}g_{33} \\
&\quad + c_{11}f_{33}g_{33} - c_{44}e_{33}\mu_{11} + 2c_{13}e_{15}\mu_{33} + c_{13}e_{31}\mu_{33} - c_{11}e_{33}\mu_{33}), \\
\alpha_{11}^{(3)} &= -[-e_{15}f_{15}f_{33} - e_{31}f_{15}f_{33} - e_{15}f_{31}f_{33} - e_{31}f_{31}f_{33} + c_{44}f_{33}g_{11} - c_{13}f_{15}g_{33} - c_{13}f_{31}g_{33} - c_{44}f_{31}g_{33} \\
&\quad + c_{11}f_{33}g_{33} + c_{13}e_{15}\mu_{33} + c_{13}e_{31}\mu_{33} + c_{44}e_{31}\mu_{33} + e_{33}(f_{15}^2 + 2f_{15}f_{31} + f_{31}^2 - c_{44}\mu_{11} - c_{11}\mu_{33})], \\
\alpha_{12}^{(3)} &= -c_{44}(f_{33}g_{33} - e_{33}\mu_{33}), \alpha_1^{(4)} = \gamma^3c_{11}(e_{15}g_{11} + f_{15}\varepsilon_{11}), \alpha_2^{(4)} = 3\gamma^2c_{11}(e_{15}g_{11} + f_{15}\varepsilon_{11}), \\
\alpha_3^{(4)} &= 3\gamma c_{11}(e_{15}g_{11} + f_{15}\varepsilon_{11}), \alpha_4^{(4)} = c_{11}(e_{15}g_{11} + f_{15}\varepsilon_{11}), \alpha_5^{(4)} = \gamma^3(-c_{13}e_{15}g_{11} - c_{13}f_{15}\varepsilon_{11}), \\
\alpha_6^{(4)} &= \gamma^2(e_{31}^2f_{15} - e_{15}e_{31}f_{31} - 3c_{13}e_{15}g_{11} - c_{13}e_{31}g_{11} + c_{11}e_{33}g_{11} + c_{11}e_{15}g_{33} - 3c_{13}f_{15}\varepsilon_{11} - c_{13}f_{31}\varepsilon_{11} \\
&\quad + c_{11}f_{33}\varepsilon_{11} + c_{11}f_{15}\varepsilon_{33}), \\
\alpha_7^{(4)} &= \gamma(e_{15}e_{31}f_{15} + 2e_{31}^2f_{15} - e_{15}^2f_{31} - 2e_{15}e_{31}f_{31} - 3c_{13}e_{15}g_{11} - 2c_{13}e_{31}g_{11} - c_{44}e_{31}g_{11} + 2c_{11}e_{33}g_{11} \\
&\quad + 2c_{11}e_{15}g_{33} - 3c_{13}f_{15}\varepsilon_{11} - 2c_{13}f_{31}\varepsilon_{11} - c_{44}f_{31}\varepsilon_{11} + 2c_{11}f_{33}\varepsilon_{11} + 2c_{11}f_{15}\varepsilon_{33}), \\
\alpha_8^{(4)} &= -\{-e_{31}^2f_{15} + e_{15}^2f_{31} + (c_{13} + c_{44})e_{31}g_{11} - c_{11}e_{33}g_{11} + e_{15}[e_{31}(-f_{15} + f_{31}) + c_{13}g_{11} - c_{11}g_{33}], +c_{13}f_{15}\varepsilon_{11} \\
&\quad + c_{13}f_{31}\varepsilon_{11} + c_{44}f_{31}\varepsilon_{11} - c_{11}f_{33}\varepsilon_{11} - c_{11}f_{15}\varepsilon_{33}\}, \\
\alpha_9^{(4)} &= \gamma^2(-e_{31}e_{33}f_{15} + e_{15}e_{31}f_{33} - c_{13}e_{15}g_{33} - c_{13}f_{15}\varepsilon_{33}), \\
\alpha_{10}^{(4)} &= \gamma(-e_{15}e_{33}f_{15} - 2e_{31}e_{33}f_{15} - e_{31}e_{33}f_{31} + e_{15}^2f_{33} + 2e_{15}e_{31}f_{33} + e_{31}^2f_{33} + c_{44}e_{33}g_{11} - 2c_{13}e_{15}g_{33} - c_{13}e_{31}g_{33} \\
&\quad + c_{11}e_{33}g_{33} + c_{44}f_{33}\varepsilon_{11} - 2c_{13}f_{15}\varepsilon_{33} - c_{13}f_{31}\varepsilon_{33} + c_{11}f_{33}\varepsilon_{33}),
\end{aligned}
$$

$$\alpha_{11}^{(4)} = e_{15}^2 f_{33} + e_{31}^2 f_{33} + c_{44}e_{33}g_{11} + c_{11}e_{33}g_{33} - e_{15}[e_{33}(f_{15}+f_{31}) - 2e_{31}f_{33} + c_{13}g_{33}] - e_{31}[e_{33}(f_{15}+f_{31}) + (c_{13}+c_{44})g_{33}] + c_{44}f_{33}\varepsilon_{11} - c_{13}f_{15}\varepsilon_{33} - c_{13}f_{31}\varepsilon_{33} - c_{44}f_{31}\varepsilon_{33} + c_{11}f_{33}\varepsilon_{33},$$

$$\alpha_{12}^{(4)} = c_{44}(e_{33}g_{33} + f_{33}\varepsilon_{33}).$$

The solving processes of the known functions $\beta_1(s)$, $\beta_2(s)$, $\beta_3(s)$, $\beta_4(s)$, $\beta_5(s)$ and $\beta_6(s)$

For $\lambda_1^2 \neq \lambda_2^2 \neq \lambda_3^2 \neq \lambda_4^2 > 0$, we can write the matrices $[X_i]$ $(i = 1, 2)$ as follows:

$$[X_1] = \begin{bmatrix} \beta_1^{(1)} & \beta_2^{(1)} & \beta_3^{(1)} & \beta_4^{(1)} \\ \chi_1^{(1)} & \chi_2^{(1)} & \chi_3^{(1)} & \chi_4^{(1)} \\ \chi_1^{(3)} & \chi_2^{(3)} & \chi_3^{(3)} & \chi_4^{(3)} \\ \chi_1^{(4)} & \chi_2^{(4)} & \chi_3^{(4)} & \chi_4^{(4)} \end{bmatrix}, \quad [X_2] = \begin{bmatrix} \beta_1^{(2)} & \beta_2^{(2)} & \beta_3^{(2)} & \beta_4^{(2)} \\ \beta_1^{(3)} & \beta_2^{(3)} & \beta_3^{(3)} & \beta_4^{(3)} \\ \beta_1^{(4)} & \beta_2^{(4)} & \beta_3^{(4)} & \beta_3^{(4)} \\ \chi_1^{(2)} & \chi_2^{(2)} & \chi_3^{(2)} & \chi_4^{(2)} \end{bmatrix} \tag{A-1}$$

From Eqs. (35) and (36), we can obtain

$$a = \frac{1}{2}[X_1]^{-1}\begin{bmatrix} \bar{f}_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \frac{1}{2}[X_2]^{-1}\begin{bmatrix} \bar{f}_2 \\ \frac{D_0\bar{f}_2}{\varepsilon_0} \\ \frac{B_0\bar{f}_2}{\mu_0} \\ 0 \end{bmatrix}, \quad b = \frac{1}{2}[X_1]^{-1}\begin{bmatrix} \bar{f}_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} - \frac{1}{2}[X_2]^{-1}\begin{bmatrix} \bar{f}_2 \\ \frac{D_0\bar{f}_2}{\varepsilon_0} \\ \frac{B_0\bar{f}_2}{\mu_0} \\ 0 \end{bmatrix} \tag{A-2}$$

where $a = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{bmatrix}$, $b = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \end{bmatrix}$. So the unknown functions A_i and B_i can be expressed as follows:

$$A_i = \frac{1}{2}\left[m_{i1}\bar{f}_1 + \left(n_{i1} + \frac{D_0}{\varepsilon_0}n_{i2} + \frac{B_0}{\mu_0}n_{i3}\right)\bar{f}_2\right], \quad B_i = \frac{1}{2}\left[m_{i1}\bar{f}_1 - \left(n_{i1} + \frac{D_0}{\varepsilon_0}n_{i2} + \frac{B_0}{\mu_0}n_{i3}\right)\bar{f}_2\right] \tag{A-3}$$

where $[m_{ij}]_{4\times4} = [X_1]^{-1}$, $[n_{ij}]_{4\times4} = [X_2]^{-1}$.

Substituting Eqs. (A-3) into Eqs. (35) and (36), we have

$$\frac{\bar{f}_1}{2}\begin{bmatrix} \beta_1^{(1)} & \beta_2^{(1)} & \beta_3^{(1)} & \beta_4^{(1)} \\ \chi_1^{(1)} & \chi_2^{(1)} & \chi_3^{(1)} & \chi_4^{(1)} \\ \chi_1^{(3)} & \chi_2^{(3)} & \chi_3^{(3)} & \chi_4^{(3)} \\ \chi_1^{(4)} & \chi_2^{(4)} & \chi_3^{(4)} & \chi_4^{(4)} \end{bmatrix}\begin{bmatrix} m_{11} \\ m_{21} \\ m_{31} \\ m_{41} \end{bmatrix} + \frac{\bar{f}_1}{2}\begin{bmatrix} \beta_1^{(1)} & \beta_2^{(1)} & \beta_3^{(1)} & \beta_4^{(1)} \\ \chi_1^{(1)} & \chi_2^{(1)} & \chi_3^{(1)} & \chi_4^{(1)} \\ \chi_1^{(3)} & \chi_2^{(3)} & \chi_3^{(3)} & \chi_4^{(3)} \\ \chi_1^{(4)} & \chi_2^{(4)} & \chi_3^{(4)} & \chi_4^{(4)} \end{bmatrix}\begin{bmatrix} m_{11} \\ m_{21} \\ m_{31} \\ m_{41} \end{bmatrix} = \begin{bmatrix} \bar{f}_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{A-4}$$

$$\frac{\bar{f}_2}{2}\begin{bmatrix} \beta_1^{(2)} & \beta_2^{(2)} & \beta_3^{(2)} & \beta_4^{(2)} \\ \beta_1^{(3)} & \beta_2^{(3)} & \beta_3^{(3)} & \beta_4^{(3)} \\ \beta_1^{(4)} & \beta_2^{(4)} & \beta_3^{(4)} & \beta_3^{(4)} \\ \chi_1^{(2)} & \chi_2^{(2)} & \chi_3^{(2)} & \chi_4^{(2)} \end{bmatrix}\begin{bmatrix} n_{11}^* \\ n_{21}^* \\ n_{31}^* \\ n_{41}^* \end{bmatrix} + \frac{\bar{f}_2}{2}\begin{bmatrix} \beta_1^{(2)} & \beta_2^{(2)} & \beta_3^{(2)} & \beta_4^{(2)} \\ \beta_1^{(3)} & \beta_2^{(3)} & \beta_3^{(3)} & \beta_4^{(3)} \\ \beta_1^{(4)} & \beta_2^{(4)} & \beta_3^{(4)} & \beta_3^{(4)} \\ \chi_1^{(2)} & \chi_2^{(2)} & \chi_3^{(2)} & \chi_4^{(2)} \end{bmatrix}\begin{bmatrix} n_{11}^* \\ n_{21}^* \\ n_{31}^* \\ n_{41}^* \end{bmatrix} = \begin{bmatrix} \bar{f}_2 \\ \frac{D_0\bar{f}_2}{\varepsilon_0} \\ \frac{B_0\bar{f}_2}{\mu_0} \\ 0 \end{bmatrix} \tag{A-5}$$

where $n_{i1}^* = n_{i1} + \frac{D_0}{\varepsilon_0}n_{i2} + \frac{B_0}{\mu_0}n_{i3}$.

So we obtain

$$\begin{bmatrix} \beta_1^{(1)} & \beta_2^{(1)} & \beta_3^{(1)} & \beta_4^{(1)} \\ \chi_1^{(1)} & \chi_2^{(1)} & \chi_3^{(1)} & \chi_4^{(1)} \\ \chi_1^{(3)} & \chi_2^{(3)} & \chi_3^{(3)} & \chi_4^{(3)} \\ \chi_1^{(4)} & \chi_2^{(4)} & \chi_3^{(4)} & \chi_4^{(4)} \end{bmatrix}\begin{bmatrix} m_{11} \\ m_{21} \\ m_{31} \\ m_{41} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{cases} \sum_{i=1}^{4}\chi_i^{(1)}m_{i1} = 0 \\ \sum_{i=1}^{4}\chi_i^{(3)}m_{i1} = 0 \\ \sum_{i=1}^{4}\chi_i^{(4)}m_{i1} = 0 \end{cases} \tag{A-6}$$

$$\begin{bmatrix} \beta_1^{(2)} & \beta_2^{(2)} & \beta_3^{(2)} & \beta_4^{(2)} \\ \beta_1^{(3)} & \beta_2^{(3)} & \beta_3^{(3)} & \beta_4^{(3)} \\ \beta_1^{(4)} & \beta_2^{(4)} & \beta_3^{(4)} & \beta_3^{(4)} \\ \chi_1^{(2)} & \chi_2^{(2)} & \chi_3^{(2)} & \chi_4^{(2)} \end{bmatrix} \begin{bmatrix} n_{11}^* \\ n_{21}^* \\ n_{31}^* \\ n_{41}^* \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{D_0}{\varepsilon_0} \\ \frac{B_0}{\mu_0} \\ 0 \end{bmatrix} \Rightarrow \sum_{i=1}^{4} \chi_i^{(2)} n_{i1}^* = 0 \tag{A-7}$$

Substituting Eqs. (A-3) into Eqs. (32) and applying Eqs. (A-6) and (A-7), we have:

$$\begin{aligned} \sigma_{zz}^{(1)}(x,0) &= \frac{e^{\gamma x}}{4\pi}\int_{-\infty}^{\infty}\sum_{i=1}^{4}\chi_i^{(1)}(s)m_{i1}\bar{f}_1(s)\,e^{isx}\,ds + \frac{e^{\gamma x}}{4\pi}\int_{-\infty}^{\infty}\sum_{i=1}^{4}\chi_i^{(1)}(s)n_{i1}^*\bar{f}_2(s)\,e^{isx}\,ds \\ &= \frac{e^{\gamma x}}{4\pi}\int_{-\infty}^{\infty}\sum_{i=1}^{4}\chi_i^{(1)}(s)n_{i1}^*\bar{f}_2(s)\,e^{isx}\,ds = \frac{e^{\gamma x}}{2\pi}\int_{-\infty}^{\infty}\beta_1(s)\bar{f}_2(s)\,e^{isx}\,ds \end{aligned} \tag{A-8}$$

$$\begin{aligned} \sigma_{xz}^{(1)}(x,0) &= \frac{e^{\gamma x}}{4\pi}\sum_{i=1}^{4}\int_{-\infty}^{\infty}\chi_i^{(2)}(s)m_{i1}\bar{f}_1(s)\,e^{isx}\,ds + \frac{e^{\gamma x}}{4\pi}\sum_{i=1}^{4}\int_{-\infty}^{\infty}\chi_i^{(2)}(s)n_{i1}^*\bar{f}_2(s)\,e^{isx}\,ds \\ &= \frac{e^{\gamma x}}{4\pi}\sum_{i=1}^{4}\int_{-\infty}^{\infty}\chi_i^{(2)}(s)m_{i1}\bar{f}_1(s)\,e^{isx}\,ds = \frac{e^{\gamma x}}{2\pi}\sum_{i=1}^{4}\int_{-\infty}^{\infty}\beta_2(s)\bar{f}_1(s)\,e^{isx}\,ds \end{aligned} \tag{A-9}$$

$$\begin{aligned} D_z^{(1)}(x,0) &= \frac{e^{\gamma x}}{4\pi}\int_{-\infty}^{\infty}\sum_{i=1}^{4}\chi_i^{(3)}(s)m_{i1}\bar{f}_1(s)\,e^{isx}\,ds + \frac{e^{\gamma x}}{4\pi}\int_{-\infty}^{\infty}\sum_{i=1}^{4}\chi_i^{(3)}(s)n_{i1}^*\bar{f}_2(s)\,e^{isx}\,ds \\ &= \frac{e^{\gamma x}}{4\pi}\int_{-\infty}^{\infty}\sum_{i=1}^{4}\chi_i^{(3)}(s)n_{i1}^*\bar{f}_2(s)\,e^{isx}\,ds = \frac{e^{\gamma x}}{2\pi}\int_{-\infty}^{\infty}\beta_3(s)\bar{f}_2(s)\,e^{isx}\,ds \end{aligned} \tag{A-10}$$

$$\begin{aligned} D_x^{(1)}(x,0) &= \frac{e^{\gamma x}}{4\pi}\int_{-\infty}^{\infty}\sum_{i=1}^{4}\chi_i^{(5)}(s)m_{i1}\bar{f}_1(s)\,e^{isx}\,ds + \frac{e^{\gamma x}}{4\pi}\int_{-\infty}^{\infty}\sum_{i=1}^{4}\chi_i^{(5)}(s)n_{i1}^*\bar{f}_2(s)\,e^{isx}\,ds, (\bar{f}_1(s)=0) \\ &= \frac{e^{\gamma x}}{4\pi}\int_{-\infty}^{\infty}\sum_{i=1}^{4}\chi_i^{(5)}(s)n_{i1}^*\bar{f}_2(s)\,e^{isx}\,ds = \frac{e^{\gamma x}}{2\pi}\int_{-\infty}^{\infty}\beta_4(s)\bar{f}_2(s)\,e^{isx}\,ds \end{aligned} \tag{A-11}$$

$$\begin{aligned} B_z^{(1)}(x,0) &= \frac{e^{\gamma x}}{4\pi}\int_{-\infty}^{\infty}\sum_{i=1}^{4}\chi_i^{(4)}(s)m_{i1}\bar{f}_1(s)\,e^{isx}\,ds + \frac{e^{\gamma x}}{4\pi}\int_{-\infty}^{\infty}\sum_{i=1}^{4}\chi_i^{(4)}(s)n_{i1}^*\bar{f}_2(s)\,e^{isx}\,ds \\ &= \frac{e^{\gamma x}}{4\pi}\int_{-\infty}^{\infty}\sum_{i=1}^{4}\chi_i^{(4)}(s)n_{i1}^*\bar{f}_2(s)\,e^{isx}\,ds = \frac{e^{\gamma x}}{2\pi}\int_{-\infty}^{\infty}\beta_5(s)\bar{f}_2(s)\,e^{isx}\,ds \end{aligned} \tag{A-12}$$

$$\begin{aligned} B_x^{(1)}(x,0) &= \frac{e^{\gamma x}}{4\pi}\int_{-\infty}^{\infty}\sum_{i=1}^{4}\chi_i^{(6)}(s)m_{i1}\bar{f}_1(s)\,e^{isx}\,ds + \frac{e^{\gamma x}}{4\pi}\int_{-\infty}^{\infty}\sum_{i=1}^{4}\chi_i^{(6)}(s)n_{i1}^*\bar{f}_2(s)\,e^{isx}\,ds \\ &= \frac{e^{\gamma x}}{4\pi}\int_{-\infty}^{\infty}\sum_{i=1}^{4}\chi_i^{(6)}(s)n_{i1}^*\bar{f}_2(s)\,e^{isx}\,ds = \frac{e^{\gamma x}}{2\pi}\int_{-\infty}^{\infty}\beta_6(s)\bar{f}_2(s)\,e^{isx}\,ds \end{aligned} \tag{A-13}$$

References

Abramowitz, M., Stegun, I.A., 1968. Handbook of Mathematical Functions. Dover, New York, p. 778.

Avellaneda, M., Harshe, G., 1994. Magnetoelectric effect in piezoelectric/magnetostrictive multiplayer (2–2) composites. Journal of Intelligent material Systems and Structures 5, 501–513.

Benveniste, Y., 1995. Magnetoelectric effect in fibrous composites with piezoelectric and magnetostrictive phases. Physical Review B 51, 16424–16427.

Chen, W.Q., Lee, K.Y., Ding, H.J., 2004. General solution for transversely isotropic magneto-electro-thermo-elasticity and the potential theory method. International Journal of Engineering Science 42, 1361–1379.

Chung, M.Y., Ting, T.C.T., 1995. The Green function for a piezoelectric piezomagnetic anisotropic elastic medium with an elliptic hole or rigid inclusion. Philosophical Magazine Letters 72, 405–410.

Dascalu, D., Homentcovschi, D., 2002. An intermediate crack model for flaws in piezoelectric solids. ACTA Mechanica 154 (1-4), 85–100.

Deeg, W.E.F., 1980. The analysis of dislocation, crack and inclusion problems in piezoelectric solids, Ph.D. Thesis, Stanford University.

Delale, F., Erdogan, F., 1988. On the mechanical modeling of the interfacial region in bonded half-planes. ASME Journal of Applied Mechanics 55, 317–324.

Ding, H.J., Chen, B., Liang, J., 1996. General solutions for coupled equations for piezoelectric media. International Journal of Solids and Structures 33 (16), 2283–2296.

Erdelyi, A., 1954. Tables of Integral Transforms, Vol. I. McGraw-Hill, New York.

Fildis, H., Yahsi, O.S., 1996. The axisymmetric crack problem in a non-homogeneous interfacial region between homogeneous half-spaces. International Journal of Fracture 78, 139–163.

Gao, C.F., Kessler, H., Balke, H., 2003a. Fracture analysis of electromagnetic thermoelastic solids. European Journal of Mechanics A – Solid 22 (3), 433–442.

Gao, C.F., Kessler, H., Balke, H., 2003b. Crack problems in magnetoelectroelastic solids. Part I: Exact solution of a crack. International Journal of Engineering Science 41 (9), 969–981.

Gao, C.F., Kessler, H., Balke, H., 2003c. Crack problems in magnetoelectroelastic solids. Part II: General solution of collinear cracks. International Journal of Engineering Science 41 (9), 983–994.

Gao, C.F., Tong, P., Zhang, T.Y., 2003d. Interfacial crack problems in magneto-electroelastic solids. International Journal of Engineering Science 41 (18), 2105–2121.

Gao, H.J., Zhang, T.Y., Tong, P., 1997. Local and global energy release rates for an electrically yielded crack in a piezoelectric ceramics. Journal of Mechanics and Physics of Solids 1997 45 (4), 491–510.

Gradshteyn, I.S., Ryzhik, I.M., 1980. Table of Integral, Series and Products. Academic Press, New York.

Hao, T.H., Shen, Z.Y., 1994. A new electric boundary condition of electric fracture mechanics and its applications. Engineering Fracture Mechanics 47 (6), 793–802.

Hao, T.H., 2001. Multiple collinear cracks in a piezoelectric material. International Journal of Solids and Structures 38 (50-51), 9201–9208.

Harshe, G., Dougherty, J.P., Newnham, R.E., 1993. Theoretical modeling of 3–0/0–3 magnetoelectric composites. International Journal of Applied Electromagnetics in Materials 4, 161–171.

Huang, J.H., Kuo, W.S., 1997. The analysis of piezoelectric/piezomagnetic composite materials containing ellipsoidal inclusions. Journal of Applied Physics 81 (3), 1378–1386.

Itou, S., 1978. Three dimensional waves propagation in a cracked elastic solid. Journal of Applied Mechanics 45 (6), 807–811.

Jin, B., Zhong, Z., 2002. A moving mode-III crack in functionally graded piezoelectric material: permeable problem. Mechanics Research Communications 29, 217–224.

Jin, D.R., 2003. Functionally graded PZT/ZnO piezoelectric composites. Journal of Materials Science Letters 22, 971–974.

Li, C.Y., Weng, G.J., 2002. Antiplane crack problem in functionally graded piezoelectric materials. Journal of Applied Mechanics 69 (4), 481–488.

Li, J.Y., 2000. Magnetoelectroelastic multi-inclusion and inhomogeneity problems and their applications in composite materials. International Journal of Engineering Science 38, 1993–2011.

Liu, J.X., Liu, X.L., Zhao, Y.B., 2001. Green's functions for anisotropic magnetoelectroelastic solids with an elliptical cavity or a crack. International Journal of Engineering Science 39 (12), 1405–1418.

McMeeking, R.M., 1989. On mechanical stress at cracks in dielectrics with application to dielectric breakdown. Journal of Applied Physics 62, 3122–3316.

Mikhailov, G.K., Parton, V.S., 1990. Electromagnetoelasticity. Hemisphere, New York.

Morse, P.M., Feshbach, H., 1958. Methods of Theoretical Physics. Vol. I. McGraw-Hill, New York.

Nan, C.W., 1994. Magnetoelectric effect in composites of piezoelectric and piezomagnetic phases. Physical Review B 50, 6082–6088.

Ozturk, M., Erdogan, F., 1997. Mode I crack problem in an inhomogeneous orthotropic medium. International Journal of Engineering Science 35, 869–883.

Pak, Y.E., 1990. Crack extension force in a piezoelectric material. Journal of Applied mechanics 57, 647–653.

Pan, E., 2002. Three-dimensional Green's functions in anisotropic magneto-electro-elastic bimaterails. Zeitschrift fur Angewandte Mathematik und Physik 53, 815–838.

Parton, V.S., 1976. Fracture mechanics of piezoelectric materials. ACTA Astronautra 3, 671–683.

Shbeeb, N.I., Binienda, W.K., 1999. Analysis of an interface crack for a functionally graded strip sandwiched between two homogeneous layers of finite thickness. Engineering Fracture Mechanics 64, 693–720.

Sih, G.C., Song, Z.F., 2003. Magnetic and electric poling effects associated with crack growth in $BaTiO_3$–$CoFe_2O_4$ composite. Theoretical and Applied Fracture Mechanics 39, 209–227.

Soh, A.K., Fang, D.N., Lee, K.L., 2000. Analysis of a bi-piezoelectric ceramic layer with an interfacial crack subjected to anti-plane shear and in-plane electric loading. European Journal of Mechanics A – Solid 19, 961–977.

Song, Z.F., Sih, G.C., 2003. Crack initiation behavior in magnetoelectroelastic composite under in-plane deformation. Theoretical and Applied Fracture Mechanics 39, 189–207.

Sosa, H., 1992. On the fracture mechanics of piezoelectric solids. International Journal of Solids and Structures 29, 2613–2622.

Spyropoulos, C.P., Sih, G.C., Song, Z.F., 2003. Magnetoelectroelastic composite with poling parallel to plane of line crack under out-of-plane deformation. Theoretical and Applied Fracture Mechanics 39 (3), 281–289.

Suo, Z., Kuo, C.M., Barnett, D.M., Willis, J.R., 1992. Fracture mechanics for piezoelectric ceramics. Journal of the Mechanics and Physics of Solids 40 (4), 739–765.

Tian, W.Y., Gabbert, U., 2004. Multiple crack interaction problem in magnetoelectroelastic solids. European Journal of Mechanics A – Solid 23, 599–614.

Van Suchtelen, J., 1972. Product properties: a new application of composite materials. Phillips Research Reports 27, 28–37.

Wang, B.L., Mai, Y.W., 2003. Crack tip field in piezoelectric/piezomagnetic media. European Journal of Mechanics A – Solid 22 (4), 591–602.

Wang, B.L., Mai, Y.W., 2004. Fracture of piezoelectromagnetic materials. Mechanics Research Communications 31 (1), 65–73.

Wang, X., Shen, Y.P., 2002. The general solution of three-dimensional problems in magnetoelectroelastic media. International Journal of Engineering Science 40, 1069–1080.

Wu, T.L., Huang, J.H., 2000. Closed-form solutions for the magnetoelectric coupling coefficients in fibrous composites with piezoelectric and piezomagnetic phases. International Journal of Solids and Structures 37, 2981–3009.

Yang, F.Q., 2001. Fracture mechanics for a Mode I crack in piezoelectric materials. International Journal of Solids and Structures 38, 3813–3830.

Zhang, T.Y., Qian, C.F., Tong, P., 1998. Linear electro-elastic analysis of a cavity or a crack in a piezoelectric material. International Journal of Solids and Structures 35, 2121–2149.

Zhang, T.Y., Tong, P., 1996. Fracture mechanics for a mode III crack in a piezoelectric material. International Journal of Solids and Structures 33, 343–359.

Zhong, Z., Meguid, S.A., 1997. Analysis of a circular arc-crack in piezoelectric materials. International Journal of Fracture 84, 143–158.

Zhou, Z.G., Wang, B., 2004. Two parallel symmetry permeable cracks in functionally graded piezoelectric/piezomagnetic materials under anti-plane shear loading. International Journal of Solids and Structures 41, 4407–4422.

Zhou, Z.G., Wang, B., Sun, Y.G., 2004. Two collinear interface cracks in magneto-electro-elastic composites. International Journal of Engineering Science 42, 1157–1167.

Zhou, Z.G., Wu, L.Z., Wang, B., 2005a. The dynamic behavior of two collinear interface cracks in magneto-electro-elastic composites. European Journal of Mechanics A – Solids 24 (2), 253–262.

Zhou, Z.G., Wu, L.Z., Wang, B., 2005b. The behavior of a crack in functionally graded piezoelectric/piezomagnetic materials under anti-plane shear loading. Archive of Applied Mechanics 74 (8), 526–535.

Zhou, Z.G., Zhang, P.W., Wu, L.Z., 2007. The closed form solution of a Mode-I crack in the piezoelectric/piezomagnetic materials. International Journal of Solids and Structures 44, 419–435.